DOCUMENT RESUME

ED 090 023 24 SE 017 593

AUTHOR Prekeges, Demitrios P.
TITLE Relationship Between Selected Teacher Variables and Growth in Arithmetic in Grades Four, Five and Six. Final Report.
INSTITUTION Eastern Washington State Coll., Cheney.
SPONS AGENCY Office of Education (DHEW), Washington, D.C. Bureau of Research.
BUREAU NO BR-9-I-030
PUB DATE Dec 73
GRANT OEG-9-70-0002
NOTE 184p.

EDRS PRICE MF-$0.75 HC-$9.00 PLUS POSTAGE
DESCRIPTORS *Achievement; Attitudes; Effective Teaching; *Elementary School Mathematics; Instruction; Mathematics Education; *Predictor Variables; *Research; *Teacher Characteristics; Teacher Experience; Teacher Influence; Teacher Qualifications; Teaching Experience; Teaching Quality

ABSTRACT

This study reports on the relationship between nine teacher-variables and student achievement. Student achievement was portioned out into four independent variables to be used in a multiple linear regression analysis; they were growth in understanding, growth in problem solving, growth in computation, and overall achievement. Tests to measure these variables were constructed by the researcher and made compatible with the content being taught. Teacher variables include knowledge test scores, attitudes toward traditional or "new math" programs, academic preparation, and principal rating scales. All test instruments are included in the appendix. The population for this study was 61 fourth-, fifth-, and sixth-grade classes and their 61 teachers, selected randomly from 400 possible classes. There were no significant relationships between any teacher variables, when taken individually or in groups, and student growth in any of the three areas--understanding, problem solving, and computation--when taken individually or in groups. (JP)

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FINAL REPORT

Project No. 9-1-030

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RELATIONSHIP BETWEEN SELECTED TEACHER VARIABLES AND GROWTH IN ARITHMETIC IN GRADES FOUR, FIVE AND SIX

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The research reported herein was performed pursuant to a grant with the Office of Education, U. S. Department of Health, Education, and Welfare. Contractors undertaking such projects under Government sponsorship are encouraged to express freely their professional judgment in the conduct of the project. Points of view or opinions stated do not, therefore, necessarily represent official Office of Education position or policy.

U. S. DEPARTMENT OF
HEALTH, EDUCATION, AND WELFARE

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ABSTRACT

THE RELATIONSHIP BETWEEN
SELECTED TEACHER VARIABLES AND GROWTH IN ARITHMETIC
IN GRADES FOUR, FIVE, AND SIX

by

Demitrios Peter Prekeges

Chairman: Professor Eric MacPherson

Problem

For many years mathematicians and mathematics educators have been stating that teachers of arithmetic need a greater knowledge in mathematics and methods of teaching mathematics. Many colleges have required more mathematics for their future elementary teachers. The belief is that an individual with more mathematics will teach more mathematics to their elementary students.

The review of the literature as a whole does not agree. Few researchers have found significant relationships between teacher knowledge and teacher effectiveness. The review of the literature further indicates that most researchers did not measure teacher variables precisely. Also, most researchers neither partitioned nor measured directly student growth. They used standardized tests or administrative ratings to determine teacher effectiveness.

Procedures

Two instruments were constructed to measure teacher understanding and teacher attitude. The test of understanding was designed to measure

the mathematical understandings as related to the arithmetic series and syllabus of the two school districts participating in this study. The attitude inventory was a forced choice inventory which measured the teacher's attitude toward contemporary mathematics as opposed to traditional mathematics. Each participating teacher also completed a questionnaire giving information about 12 other commonly reported variables. These were in the areas of quarter hours of college mathematics, quarter hours of new mathematics, quarter hours of mathematics methods, experience, and principal's ratings as he viewed the teachers.

To determine teacher effectiveness, student tests were constructed to directly measure the material of the arithmetic series and syllabus of the two school districts participating in this study. Three tests were constructed for each grade level; an understanding test, a problem solving test, and a computation test. The pre-test post-test procedure was used to determine student growth.

The population for this study was 61 fourth, fifth, and sixth grade classes and their 61 teachers. The population was randomly selected from over 400 teachers in two Washington State school districts. Both districts used the same arithmetic series, similar syllabus, and are in different geographic locations.

Results and Conclusions

There were no significant relationships between any of the teacher variables, when taken individually or in groups, and student growth in any of the three areas--understanding, problem solving, and computation--when taken individually or in groups.

In this study, every effort was made to eliminate the deficiencies of previous studies. Yet their results are, in general, confirmed. If mathematicians and mathematics educators are to persist in their opinion that teacher change begets student gain, then it seems that different independent variables must be identified. It seems highly unlikely that success would reward any further exploration of those identified in this study.

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Chapter 1

INTRODUCTION

The Problem

For many years mathematicians and mathematics educators have been stating that teachers of arithmetic need a greater knowledge of mathematics. Morton,[1] in 1939, recommended that every elementary teacher be required to complete 6 to 10 semester hours in mathematics. Wren,[2] in 1941, pointed out that the mathematical background of teachers was inadequate and it was up to the teacher training colleges to improve the situation.

In the 10 year period following World War II, many organizations emphasized the mathematical needs of the elementary teachers. The first such postwar suggestion was made by the Commission on Postwar Plans.[3] This commission recommended that teachers of arithmetic should study a course in the teaching of arithmetic and one or more courses in subject matter background. This report was followed by the Manpower

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[1] R. L. Morton, "Mathematics in the Training of Arithmetic Teachers," Mathematics Teacher, 32:106-110, December, 1939.

[2] F. L. Wren, "Questions for the Teacher of Arithmetic," Arithmetic in General Education, Sixteenth Yearbook of the National Council of Teachers of Mathematics (Washington, D.C.: National Council of Teachers of Mathematics, 1941), pp. 290-303.

[3] Commission on Postwar Plans (Washington, D.C.: National Council of Teachers of Mathematics, 1945).

Report[4] which recommended, "A professionalized subject-matter course emphasizing the use of mathematics in projects undertaken by children to learn the meaning of concepts is a minimum requirement."[5] More recently the Mathematical Association of America, through its Committee on the Undergraduate Program in Mathematics,[6] recommended that every teacher of arithmetic should have a minimum of three college courses in mathematics consisting of four semesters of mathematics and a one-semester methods course in arithmetic. The Committee also suggested content outlines for these courses.

During the same period of time many contemporary mathematicians and mathematics educators expressed their viewpoint. In 1948, Wren[7] again wrote on the needs of the elementary teacher. He pointed out that functional competence in arithmetic is essential as a characteristic of the educated individual. He indicated that functional competence in arithmetic consisted of:

1. Proficiency in fundamental skills
2. Comprehension of basic concepts
3. Appreciation of significant meanings
4. Development of desirable attitudes
5. Efficiency in making sound applications

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[4] Manpower for Research, Scientific and Public Policy, Vol. IV (Washington, D.C.: Government Printing Office, 1947).

[5] Ibid., p. 11.

[6] Recommendations for the Training of Teachers, A Summary (Buffalo, New York: Mathematics Association of America, 1961).

[7] F. L. Wren, "The Professional Preparation of Teachers of Arithmetic," Supplementary Educational Monographs, No. 66 (Chicago: University of Chicago Press, 1948), pp. 80-90.

6. Confidence in making intelligent and independent interpretations.[8]

Wren then considered these as the six major objectives in arithmetic. Surely, if these are the objectives, a teacher must have these competencies. In 1949, Layton[9] surveyed the certification requirements of mathematics teachers and found that most states did not have any requirements. He recommended the development of such requirements. In 1949, Glennon[10] tested a group of college freshmen and seniors and found that the mathematical understanding of the freshmen was higher than the mathematical understanding of the seniors. These results seem to indicate that either a loss of mathematical understanding takes place while a student is in college, or the freshmen in 1949 were better prepared than the seniors. In 1951, Layton[11] surveyed the training prescribed by teacher training colleges and found that a majority did not require any mathematics courses for their elementary teachers. In 1951, Newsom[12] outlined the mathematical background he felt was needed by elementary teachers. This outline included topics

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[8] Ibid., p. 82.

[9] W. I. Layton, "The Certification of Teachers of Mathematics," Mathematics Teacher, 42:377-380, May, 1949.

[10] V. J. Glennon, "A Study in Needed Redirection in the Preparation of Teachers of Arithmetic," Mathematics Teacher, 42:389-396, December, 1949.

[11] W. I. Layton, "Mathematical Training Prescribed by Teachers Colleges in Preparation of Elementary Teachers," Mathematics Teacher, 44:551-556, May, 1951.

[12] C. V. Newsom, "Mathematical Background Needed by Teachers," The Teaching of Arithmetic, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), pp. 232-250.

such as historical development, the real number system, measurement, and applications. In 1951, Grossnickle[13] surveyed the state teacher training colleges and received responses from 129 of them. He found that more colleges were requiring four years of study to teach than was the case 20 years earlier. He also found little change in the mathematics requirements of future elementary teachers. He recommended that all future teachers should have a methods course in the teaching of mathematics and those who had not taken mathematics beyond the eighth grade should have a content course in mathematics before their methods course.

In 1953, mathematics educators were still discussing these same problems. Schaaf,[14] after writing about the lack of courses for teachers, outlined the scope of a course dealing with the subject matter of arithmetic. He suggested that all future teachers need such a course. Phillips[15] attempted to show the need for a mathematics course for elementary teachers by recording data about students entering the course "Arithmetic for Teachers" at the University of Illinois. Two of his seven conclusions were:

1. The four major factors influencing the students' reaction to mathematics are method of presentation, opportunitites for achievement, teacher's personality, and type of problems solved.

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[13] F. C. Grossnickle, "The Training of Teachers of Arithmetic," The Teaching of Arithmetic, Fiftieth Yearbook of the National Society for the Study of Education, Part II (Chicago: University of Chicago Press, 1951), pp. 203-231.

[14] W. L. Schaaf, "Arithmetic for Arithmetic Teachers," School Science and Mathematics, 53:537-543, October, 1953.

[15] C. Phillips, "Background and Mathematical Achievement of Elementary Education Students in Arithmetic for Teachers," School Science and Mathematics, 53:48-52, January, 1953.

2. Achievement in the meaning and understanding of arithmetic is extremely low.[16]

Orleans and Wandt[17] wrote:

> If arithmetic is to be taught so that children acquire real understanding of arithmetic processes and concepts, it would seem obvious that the teachers of arithmetic must possess the understanding that they are attempting to transmit to their students.[18]

They then presented the findings of Orleans[19] from a study in which he administered a test to 722 subjects. The purpose of the evaluation was to determine the understanding of the processes and concepts of arithmetic. He concluded that future and practicing teachers have a low understanding of these processes and concepts. Orleans and Wandt then wrote:

> If the understanding of arithmetic possessed by teachers is to be increased, teacher-training institutions must make this one of their goals. The teacher-education institutions may have only an indirect influence on the program of number work in the schools, but they can directly influence the prospective teacher's knowledge and understanding of arithmetic and his preparation for his responsibilities in getting children to learn about numbers.[20]

In 1956, Snader,[21] after reviewing the literature, wrote, "This situation is deplorable, to say the least."[22] He sent a questionnaire

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[16] Ibid., p. 51.

[17] J. S. Orleans and E. Wandt, "The Understanding of Arithmetic Possessed by Teachers," Elementary School Journal, 53:501-507, May, 1953.

[18] Ibid., p. 501.

[19] J. S. Orleans, The Understanding of Arithmetic Processes and Concepts Possessed by Teachers of Arithmetic, Publication No. 12 (New York: Office of Research and Evaluation, College of the City of New York, 1952).

[20] Orleans and Wandt, op. cit., p. 507.

[21] D. Snader, "Mathematical Background for Teachers of Arithmetic," The Arithmetic Teacher, 3:59-65, March, 1956.

[22] Ibid., p. 61.

to a representative group of specialists in arithmetic and found this group would like elementary school teachers to have studied mathematics for a minimum of six semester hours. Further, the mathematics studied should not be the typical college mathematics, but it should be mathematics that involves mainly the understanding of the backgrounds needed by a teacher of arithmetic.

Such findings, statements, and recommendations have led to the development of courses for future elementary teachers. Most colleges now have at least one such required course and some colleges[23] have established a major emphasis in mathematics for elementary teachers. These are generally classes designed for this purpose as distinct from regular freshman and sophomore mathematics programs.

The recommendations for and the extensive development of these courses are based on the belief that if the teacher has a better mathematical background, his students will learn and understand more arithmetic. Metzner[24] has been one of the few to question this belief. In his summary of a symposium at the Harvard Graduate School of Education[25] he quotes Professor James Coleman of John Hopkins University: ". . . no one knows enough about teachers' performance to be able to predict the effects of longer teacher preparation on pupil achievement."[26] A review of educational research literature seems to support the views of Metzner and Coleman where the elementary teacher is involved. In fact,

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[23]Eastern Washington State College, Cheney, Washington; Southwestern State College, Weatherford, Oklahoma; Northern Michigan University, Marquette, Michigan.

[24]S. Metzner, "The Teacher Preparation Myth: A Phoenix Too Frequent," Phi Delta Kappan, L:105-107, October, 1968.

[25]Ibid. [26]Ibid., p. 105.

as is discussed in Chapter 2, this researcher failed to find any studies which, with any confidence, imply that increased mathematical education of elementary teachers or future elementary teachers increases pupil achievement.

As will be discussed below, it is questionable that as much confidence as the results indicate can be placed on these studies.

At the same time, it is evident the majority of the people working in mathematics education feel that better training for the elementary teacher is a necessity and this training should include more work in mathematics which should be designed to teach mathematical understanding. These same experts seem to feel that such training will lead to better pupil achievement in mathematics even though there is very little good evidence to substantiate this belief.

Some studies have been carried out in an effort to determine whether or not teacher knowledge has an effect on student achievement. Two things characterize most of these studies:

1. Indirect measures of teacher ability. Most of these studies use college credits in mathematics or some type of arithmetic test. They do not attempt to measure mathematical understanding.

2. Imprecise measures of student performance. Most of these studies use some type of published standardized test. They do not attempt to measure the material of a given text or the material of the school syllabus.

The naive belief that increased teacher understanding has an effect on student understanding remains strong. As the review of the literature indicates in Chapter 2, there is certainly no hope of supporting this belief by replicating or expanding on the studies with

the two failings noted above. By constructing tests which carefully measure teacher understanding in and attitude toward mathematics and student competencies in mathematics, it might be possible to identify some relationship between these variables.

In this study, a very serious attempt is made to identify and measure precisely those teacher variables most apt to be related to similarly identified and measured student variables. This study first tests certain a priori hypotheses concerning the relationship between these variables, and then searches, speculatively, for unexpected possible relationships which might form a foundation for further study.

Chapter 2

BACKGROUND AND JUSTIFICATION

This review of the literature is separated into three sections. The first section deals with secondary (including junior high school) teachers' knowledge and how it is related to student achievement; the second section deals with the elementary (grades Kindergarten through six) teachers' knowledge and how it is related to student achievement; and the third section deals with teachers' attitude toward mathematics.

Secondary School Reviews

The first postwar report which attempted to determine a relationship between teacher variables and student growth seems to be Rostker's[1] report of the results of data collected in 1936 and 1937. Rostker tested 350 social studies students in the seventh and eighth grades who were taught by 28 different teachers. He pre-tested and post-tested the students and used student gains as a measure of teaching ability. He measured the teachers' subject matter knowledge by using tests covering the material taught. He wrote:

> These teacher measures are primarily tests of information and indicate no significant relationship between knowledge of subject information and teaching ability.[2]

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[1] L. E. Rostker, "The Measurement of Teaching Ability, Study Number One," The Journal of Experimental Education, 14:6-51, September, 1945.

[2] Ibid., p. 45.

Rolfe,[3] using the same age group but using citizenship as the subject matter material, found similar results. La Duke,[4] also using seventh and eighth grade students but using the child's sense of responsibility in the functioning of a democratic society, found similar results. Both Rolfe and La Duke collected their data between 1937 and 1939, but did not report their results until 1945.

In 1946, Lins[5] attempted to determine whether or not a relationship exists between pre-service education and student gains. His sample consisted of 17 first year teachers and their 27 classes, which comprised most areas and levels of the secondary schools. The teacher measures for pre-service education were grades in college courses and ratings of possible success in teaching by their college professors. The student gains were calculated by pre-testing and post-testing the course material for the second semester using standardized tests. Lins found that grades and ratings in pre-service education, including practice teaching, are not significantly related to teaching efficiency as measured by student gain scores.

In 1949, Snider[6] looked at several factors which might be related to student achievement in college. He found a positive but not

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[3] J. F. Rolfe, "The Measurement of Teaching Ability, Study Number Two," The Journal of Experimental Education, 14:52-74, September, 1945.

[4] C. V. La Duke, "The Measurement of Teaching Ability, Study Number Three," The Journal of Experimental Education, 14:75-100, September, 1945.

[5] L. J. Lins, "The Prediction of Teaching Efficiency," The Journal of Experimental Education, 15:2-60, September, 1946.

[6] H. L. Snider, "Relationships Between Factors of High School Background and Achievement in Certain Subject Fields" (unpublished Doctoral dissertation, University of Nebraska, 1949).

significant relationship between the college preparation secondary teachers have in their teaching field and the achievement of their students in this field when the students attend college.

It would seem that the studies reported in the forties did nothing to support the premise that teacher change causes student change of a like kind. It might well be, as noted in Chapter 1, that teacher variables and student variables were not adequately measured by the standardized tests and other procedures used in these studies.

In 1950, Schunert[7] compared the final achievement of algebra and geometry classes whose teachers had less than two years of college mathematics with algebra and geometry classes whose teachers had more than two years of college mathematics. He found no significant difference, but the results favored those teachers with the lesser amount of college preparation in mathematics.

In 1957, Taylor,[8] attempting to find a significant relationship between teacher factors and science students, tested more than 1500 science students with the Essential High School Content Battery and the California Occupational Interest Inventory. He compared these results with four teacher factors: (1) attitude, (2) college credit in professional education, (3) college credit in science, and (4) years of experience. None of the factors had a significant relationship with

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[7] J. R. Schunert, "The Association of Mathematics Achievement with Certain Factors Resident in the Teacher, in the Teaching, in the Pupil, and in the School" (unpublished Doctoral dissertation, University of Minnesota, 1951).

[8] T. W. Taylor, "A Study to Determine the Relationship Between Growth in Interest and Achievement of High School Science Students and Science-Teacher Attitudes, Preparation, and Experience" (unpublished Doctoral dissertation, North Texas State College, 1957). Dissertation Abstracts, 17:2943-2944, No. 12, 1956/57.

the development of greater science achievement. When all four factors were taken as a composite, a significant positive relationship was found. This relationship might indicate that an interaction of factors is involved in affecting science achievement.

Sparks,[9] using the Iowa Test of Educational Development, tested a group of high school students in 1955 and again in 1958. From these results he defined high achievement schools and low achievement schools. He found that teachers in high achievement schools had taken more hours of mathematics as undergraduates in college than the teachers in low achievement schools. He also found that the students in the high achievement schools rated their teachers higher in subject matter knowledge and teacher competency than did the students in low achievement schools.

The two studies in the latter half of the fifties seem to indicate that there was some relationship between student achievement and some composite of teacher factors. Again it might be that the use of standardized tests did not give a sufficiently precise measure of student achievement. Some other evaluation more directly connected to the material to be learned might produce more positive relationships.

In 1960, Stoneking[10] attempted to determine which of the four factors--age, amount of teaching experience, level of academic preparation, or mathematics background--contributes most to an individual's understanding of selected basic arithmetical principles and

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[9] J. N. Sparks, "A Comparison of Iowa High Schools Ranking High and Low in Mathematical Achievement" (unpublished Doctoral dissertation, University of Iowa, 1960).

[10] L. W. Stoneking, "Factors Contributing to Understanding of Selected Basic Arithmetical Principles and Generalizations" (unpublished Doctoral dissertation, Indiana University, 1960).

generalizations. He administered his self-constructed instrument to measure basic arithmetical principles and generalizations to 1066 examinees. He also obtained a personal data sheet from each examinee to determine which of the four factors they possessed. The examinees were pupils in grades 8 through 12, students in a college preparatory course, and practicing teachers. He found that there was no significant difference in the scores of the examinees who were practicing teachers and those who were not practicing teachers. This would indicate that experience as a teacher does not enhance one's understanding of basic arithmetical principles and generalizations. These results might further indicate that experience as a teacher does not enable one to be a more effective teacher of mathematics.

In 1960, Lindstedt[11] compared the scores on the final examination of ninth grade mathematics students with the number of college mathematics courses taken by their teachers. There was no significant difference in the scores of students taught by teachers classified on the basis of the amount of mathematics preparation.

In 1962, Leonhardt,[12] using the Cooperative General Mathematics Test for High School Classes with tenth grade geometry classes, ranked 45 different high schools. The ranking was from high to low depending upon the mean score of the students. He then chose 12 schools for his analysis: four small schools, four medium sized schools, and four large

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[11] S. A. Lindstedt, "Teacher Qualification and Grade IX Mathematics Achievement," The Alberta Journal of Education, 6:76-85, June, 1960.

[12] E. A. Leonhardt, "An Analysis of Selected Factors Related to High and Low Achievement in Mathematics," (unpublished Doctoral dissertation, University of Nebraska, 1962).

schools. Two schools in each group were high-ranked and two schools in each group were low-ranked. He chose one teacher from each school. He found that more of the teachers from high-ranked schools had their major undergraduate preparation in mathematics than did those from low-ranked schools. The ratio was four teacher to three teacher. He also reported that more of the teachers from high-ranked schools had taken graduate work in mathematics than had the teachers from low-ranked schools. The ratio was two teachers to one teacher.

In analyzing the above study and similar studies, an important possible compounding variable must be noted. There could be an automatic selection process operating where the schools which are noted for their strong programs attract candidates with stronger backgrounds. It is also possible that these same people desire to continue these programs and their education; therefore, they attend graduate school to become better prepared.

In 1963, Garner[13] pre-tested and post-tested ninth grade algebra students using the Cooperative Algebra Test, Form 1. From the supervisors of the teachers of these algebra students he obtained the number of hours of college mathematics each of these teachers had taken. He found a significant relationship between the college mathematics preparation of the teachers and their pupils' achievement in algebra.

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[13] M. V. Garner, "A Study of the Educational Backgrounds and Attitudes of Teachers Toward Algebra as Related to the Atitutdes and Achievements of Their Anglo-American and Latin-American Pupils in First-Year Algebra Classes of Texas" (unpublished Doctoral dissertation, North Texas State University, 1963). Dissertation Abstracts, 24:189, No. 1, 1963.

In 1964, Peskin[14] reported a significant correlation between teacher understanding and student achievement. Teacher understanding was measured for the 55 teachers by Glennon's[15] Test of Basic Mathematical Understandings. Student achievement for the 565 students was measured by the Cooperative Arithmetic Test, Form A and some self-made tests related to the material covered.

Also in 1964, Smith[16] reported on the results of data collected in 1957-58 concerning the relationship between teacher professional education and student achievement. He used as his student criterion the results of the California Achievement Test in Arithmetic, Intermediate Battery, which he administered to 528 students in the eighth grade. The information on the 28 teachers used in this study was obtained from personnel records of the schools involved. He found a significant relationship between the credits earned in professional education courses (more than 28 credits against less than 28 credits) and student achievement as measured by the California Achievement Test in Arithmetic. He further reported that the number of college credits in mathematics and the number of years of teaching experience did not appear to be related to student achievement.

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[14] A. S. Peskin, "Teacher Understanding and Attitude and Student Achievement and Attitude in Seventh Grade Mathematics" (unpublished Doctoral dissertation, New York University, 1964).

[15] V. J. Glennon, "A Study of the Growth and Mastery of Certain Basic Mathematical Understandings on Seven Educational Levels" (unpublished Doctoral dissertation, Harvard University, 1948).

[16] R. W. Smith, "The Achievement of Eighth Grade Students in Arithmetic with Respect to Selected Patterns of Teacher Preparation" (unpublished Doctoral dissertation, University of Oklahoma, 1964).

In 1965, Goldberg et al.[17] studied 51 seventh grade classes and their 1477 pupils in the Talented Youth Project. By the end of the ninth grade, normal attrition had reduced the numbers to 37 classes and 868 students. Teacher factors such as amount of mathematical preparation, degrees earned, and experience in teaching mathematics were found to bear a significant relationship to pupil success at the end of the seventh grade. In aggregate, such factors accounted for about 20 percent of the variance in pupil achievement. However, at the end of the ninth grade, teacher factors appeared to be exerting less influence on pupil achievement than in earlier grades. When initial pupil differences for the ninth graders were controlled, the observed differences were no longer significant.

In 1967, Rouse[18] studied the correlation between the academic preparation of teachers of arithmetic and the arithmetic achievement of their students in kindergarten through grade eight. He measured the academic preparation of the teachers by totaling the mathematics courses they had taken in high school, in college, and any in-service courses. He called this total the total mathematics preparation of the teacher. The measure of student arithmetic achievement was his arithmetic scores on the California Achievement Tests. The sample was

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[17] M. L. Goldberg et al., "A Comparison of Mathematics Preprograms for Able Junior High School Students," Summary, Conclusions and Implications, Teachers College, Columbia University, New York: 1965.

[18] W. M. Rouse, Jr., "A Study of the Correlation Between the Academic Preparation of Teachers of Mathematics and the Mathematics Achievement of Their Students in Kindergarten through Grade Eight" (unpublished Doctoral dissertation, Michigan State University, 1967).

206 students and 273 teachers who had taught these students from kindergarten through grade eight. He found a low negative correlation between student achievement in both arithmetic reasoning and arithmetic fundamentals and the total mathematics preparation of the teachers responsible for their arithmetic instruction from kindergarten through the middle of grade eight.

The studies of the sixties seem to have added little to the knowledge of the relationship existing between teacher knowledge in mathematics and their students' knowledge in mathematics. It might be that the teacher variable related to teacher knowledge cannot accurately be measured by looking at the number of courses taken in preservice education. It might be better to measure teacher knowledge in some more direct way. Peskin[19] did this and did get a significant relationship. A second reason for finding very few significant relationships between teacher variables and student growth could be that most researchers use nationally standardized evaluation instruments to measure student growth. More positive results might be possible if the student achievement evaluation instrument covered that material which was pertinent to that grade. Again, Peskin[20] used some of these for the measure of student achievement and did get a significant relationship.

To summarize, using the mainly indirect techniques of these studies, there is little evidence to indicate a relationship between teacher knowledge in mathematics and student achievement in the secondary school (grades 7 through 12). Some studies do indicate that

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[19]Peskin, loc. cit.

[20]Ibid.

some student achievement may be related to some teacher knowledge, but precisely what is related to what is not indicated.

Elementary School Reviews

Many studies attempting to relate teacher knowledge to teacher effectiveness have been done at the elementary level. Few attempt to relate teachers' knowledge in a particular subject (say arithmetic) to student improvement or gain in that subject. As was noted for the secondary studies, these studies use mainly indirect measures of teacher quality and vague measures of student performance. Those that seem to be relevant to this study have been published since 1950.

The first such study was carried out by Ryans[21] in 1951. He worked with 275 teachers in the third and fourth grades. He found no significant relationship between the amount of college training (in total, no particular subject area) and a composite evaluation of effectiveness as a teacher. Three trained observers working independently determined, by observation, the effectiveness of the teacher. Notice again that Ryans used total hours of college training as the measure of his teacher variable. Further, he used the opinions of observers as his measure of effective teaching. If effective teaching means student learning, and most educators accept this definition, then one must measure the student learning and not attempt to infer it. In any case, it is not difficult to see how he could have failed to

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[21] D. G. Ryans, "A Study of the Extent of Association of Certain Professional and Personal Data with Judged Effectiveness of Teacher Behavior," The Journal of Experimental Education, 20:67-77, September, 1951.

determine a relationship between these measures of teacher knowledge and teacher effectiveness.

In contrast, Mork,[22] in 1953, constructed five different science tests for the students in 8 grade five and grade six classes. He pre-tested and post-tested these students for two consecutive years. During the second year of the study, four of the teachers (the experimental group) participated in a one year in-service program while the other four teachers (the control group) did not. The in-service course dealt with objectives, content, methods, and materials of science instruction. The course met once a month. Some of the gains on the five different tests were significant when the results of the second year were compared with the results of the first year. Because of this, Mork concluded:

> The null hypothesis was rejected with sufficient frequency to indicate that teachers, through the given test results of their pupils, show an increased effectiveness in instruction which is associated with an in-service science education program.[23]

In 1955, Steinbrook,[24] attempting to determine a relationship between college preparation and teacher effectiveness, received from administrative personnel a list of 50 teachers who were considered to have had outstanding teaching success and 50 teachers who were

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[22]G. M. A. Mork, "Effects of an In-Service Teacher Training Program on Pupil Outcomes in Fifth and Sixth Grade Science" (unpublished Doctoral dissertation, University of Minnesota, 1953). Dissertation Abstracts, 13:522-523, No. 4, 1953.

[23]Ibid., p. 523.

[24]R. S. Steinbrook, "Study of Some Differences in Background, Attitude, Experience, and Professional Preparation of Selected Elementary Teachers with Contrasting Local Success Records" (unpublished Doctoral dissertation, Indiana University, 1955). Dissertation Abstracts, 15:1013, No. 6, 1955.

considered to have had the least successful teaching experiences. He sent each of these 100 teachers a questionnaire asking for a wealth of data. His opinion of the data indicated that the total amount of college work appears to contribute to teaching effectiveness, but teaching effectiveness at the elementary school appears to be more closely related to the types of professional preparation experienced by teachers.

In 1957, Soper[25] found significant results contrary to Steinbrook.[26] Soper worked with 2656 students and 128 teachers in the fourth, fifth, and sixth grades. He separated the teachers into two groups using as his criterion for separation the amount of general academic and professional training each teacher had accumulated. He also pre-tested and post-tested their students using the Stanford Achievement Test. He found that the students with the higher gains had teachers from the group with less training. It should be noted that Soper measured teacher effectiveness by evaluating student learning. He did not depend upon the opinions of administrative personnel as did Steinbrook.

In 1959, McCall and Krause[27] worked with 73 teachers and their sixth grade students. They defined teacher effectiveness as growth

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[25]E. F. Soper, "A Study of the Relationship Between Certain Teacher-School Characteristics and Academic Progress, As Measured by Selected Standardized Tests, of Elementary Pupils in Grades Four, Five, and Six of New York State Public Schools in Cities under 10,000 Population" (unpublished Doctoral dissertation, Syracuse University, 1956).

[26]Steinbrook, loc. cit.

[27]W. A. McCall and G. R. Krause, "Measurement of Teacher Merit for Salary Purposes," The Journal of Educational Research, 53:73-75, October, 1959.

in the nine R's--reading, ritin, rithmetic, research, reasoning, reporting, relationship of persons, recreation, reasonable work skills--measuring this growth by pre-testing and post-testing the students. These results were statistically analyzed and each of the 73 teachers was given a teacher effectiveness score which ranged from least effective, 20, to most effective, 88. They found that the teachers' knowledge of a particular subject produced zero correlation when compared to teacher effectiveness. They also observed that classes taught by teachers whose average college grades were below 90 percent achieved better growth than did classes whose teachers' average college grades were above 90 percent.

In 1959, Smail[28] reported on what I would consider the most complete study to date. He worked with 97 teachers and their 2438 students in grades four, five, and six. He defined teacher effectiveness as student gain by pre-testing in the fall and post-testing in the spring. He used the arithmetic tests of The Iowa Tests of Basic Skills as his measurement instrument. He called this difference the pupils' mean-gain in arithmetic. He did not find a significant difference in pupils' mean-gain in arithmetic when the classes of teachers with two years of preparation were compared with classes of teachers with four years of preparation. However, Smail did find a significant positive relationship between the number of mathematics methods courses completed by the teacher with four years of preparation and the pupils' mean-gain in arithmetic. Smail determined teacher understanding in mathematics by

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[28] R. W. Smail, "Relationships Between Pupil Mean-Gain in Arithmetic and Certain Attributes of Teachers" (unpublished Doctoral dissertation, University of South Dakota, 1959).

administering Glennon's[29] Test of Basic Mathematical Understandings. He found that teacher understanding of basic mathematical concepts as measured by Glennon's test and pupils' mean-gain was not significant. Further, he did not find a significant relationship between the number of college mathematics courses a teacher had completed and pupil mean-gain in arithmetic. This study would indicate that an arithmetic methods course in the pre-service education of future teachers is the most important course leading to teacher effectiveness when defined as student learning.

In 1960, Barnes, Cruickshank, and J. Foster[30] used principals' ratings of the teachers' mathematics instruction as the criterion for teacher effectiveness in teaching mathematics. Their subjects were all of the fourth grade teachers from 66 different buildings. No significant relationship was found between the number of high school mathematics courses completed by the teachers and the principals' ratings as to their effectiveness in teaching mathematics. They also reported no significant relationship between the number of college mathematics courses completed by the teachers and the principals' ratings as to their effectiveness in teaching mathematics.

In 1960, Bassham[31] conducted a study somewhat similar to Smail's.[32] He tested 28 sixth grade teachers using Glennon's Test of Basic

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[29]Glennon, loc. cit.

[30]K. Barnes, C. Cruickshank, and J. Foster, "Selected Educational and Experience Factors and Arithmetic Teaching," The Arithmetic Teacher, 7:418-420, December, 1960.

[31]H. C. Bassham, "Relationship of Pupil Gain in Arithmetic Achievement to Certain Teacher Characteristics" (unpublished Doctoral dissertation, University of Nebraska, 1960).

[32]Smail, loc. cit.

Mathematical Understandings. The teachers' score on this test was considered as an indication of the level of the teachers' understanding of arithmetic. The 620 students were pre-tested in September by the California Achievement Test, Arithmetic, 1951, Form AA. The results of this test, in conjunction with other data, allowed Bassham to predict the score of each student on Form BB of the same test when it was given as a post-test in April. Any results that varied from the predicted score was called the deviation score of pupil gain. A significant relationship between teacher scores on the paper and pencil test and deviation scores of pupil gain was reported. Bassham reported that teacher understanding as measured by Glennon's test explained approximately one-fourth of the variation in the deviation scores of the pupils. He also reported that the significant relationship between teacher understanding and deviation scores existed for pupils with above mean intelligence, but not for students with below mean intelligence.

In 1960, another study of a similar nature was conducted by Heil, Powell, and Feifer.[33] The subjects in this study were 55 teachers and their fourth, fifth, and sixth graders. This study was not restricted to mathematics, but it compared teacher knowledge with student achievement in the liberal arts. The liberal arts knowledge of the teacher was measured by the Teacher Education Examination. Two parts of the examination, Professional Education Knowledge and Liberal Arts Knowledge, were administered. Student achievement was measured by

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[33] L. M. Heil, M. Powell, and I. Feifer, "Characteristics of Teacher Behavior and Competency Related to Achievement of Different Kinds of Children in Several Elementary Grades," New York: Brooklyn College, 1960.

pre-testing and post-testing with the Stanford Elementary and Intermediate Achievement Batteries. Their findings are similar to those of most other investigators; that is, negligible correlation between student achievement and teacher knowledge. They also reported negligible correlation between student achievement and the teaching effectiveness of the teachers as determined by observers.

It should be noted that all three similar studies, Smail,[34] Bassham,[35] and Heil et al.,[36] used paper and pencil tests for their measurements of teacher knowledge and standardized tests for determining student gain. Again, one must ask, "Do the paper and pencil tests taken by the teachers really measure understanding in mathematics?" One must also again question the use of a standardized test to measure pupil knowledge. "Do these tests really evaluate the syllabus of a school at the given grade level?" If the answer is no to either one or both of these questions, then it is possible that enough information is being lost to eliminate the possibility of significant differences.

In a related study, Houston,[37] in 1961, found by using objective tests that there is no difference in change in mathematics achievement and mathematics interest between two groups of fourth, fifth, and sixth grade pupils. One group of pupils had teachers who participated in an in-service education series by television while the other group of pupils had teachers who participated in a face-to-face lecture-discussion in-service education series. It seems that these results

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[34] Smail, loc. cit.

[35] Bassham, loc. cit.

[36] Heil et al., loc. cit.

[37] W. [illegible] Houston, "Selected Methods of In-Service Education and the Mathematics Achievement and Interest of Elementary School Pupils" (unpublished Doctoral dissertation, University of Texas, 1961).

were to be expected. If researchers are hardpressed to relate teacher knowledge to student gains, then it would seem even more likely that no significant relationships would be found in a study of this sort.

In a continuation of the above study, Houston and DeVault,[38] in 1963, reported that teacher growth increased student growth. They reported a significant relationship between teachers' growth in the understanding of the mathematics concepts of the in-service education program and pupils' growth in the understanding of those mathematics concepts specifically developed in this program. The researchers constructed the instruments to measure teachers' growth and pupils' growth. These instruments were designed to measure the mathematics emphasized in the in-service education program. They administered these instruments to both teachers and students as pre-tests and post-tests. They also reported no significance when teacher scores on the pre-test were compared with pupils' growth.

The above study seems to indicate that teacher growth in a given area begets student growth in that area. It also indicates that initial teacher knowledge does not relate to student growth. It should be noted that the instruments were constructed by the researchers to evaluate specific objectives. These evaluations led to the reported significant difference. It might be that if researchers are to find significant relationships, they must develop their own specific instruments to evaluate specific objectives instead of using standardized tests.

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[38] W. R. Houston and M. V. DeVault, "Mathematics In-Service Education: Teacher Growth Increases Pupil Growth," The Arithmetic Teacher, 9:243-247, May, 1963.

In 1964, Hall[39] compared the gain of students taught by 17 first year certified teachers with the gain of students taught by 21 college graduates with provisional certificates in their first teaching assignment. Student gain scores were derived from the school's administration of the Stanford Achievement Tests each September. The gain is the difference in grade level as calculated from the results of the test from one September to the next September. The six areas of the test are: (1) paragraph meaning, (2) word meaning, (3) spelling, (4) language, (5) arithmetic reasoning, and (6) arithmetic computation. The results favored the certified teachers in all of the six areas, and some of the results were significant. Hall found, as had Smith[40] and others, that there is a significant relationship between the amount of professional teacher education completed by a teacher and student achievement. In this instance, he found a significant relationship existing between professional teacher education and each of the three areas: (1) paragraph meaning, (2) word meaning, and (3) spelling. The other three areas had a positive, nonsignificant relationship with professional teacher education. The question that must be asked about this study is, "How much loss takes place during the summer and is it greater in one area than another area?"

In 1964, Watts[41] pre-tested 2121 sixth grade pupils using the California Achievement Test, Elementary. He then used a regression

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[39] H. O. Hall, "Professional Preparation and Teacher Effectiveness," The Journal of Teacher Education, 15:72-76, March, 1964.

[40] Smith, loc. cit.

[41] G. D. Watts, "A Correlation Analysis Between Level of Achievement and Certain Teacher Characteristics in Selected School Systems" (unpublished Doctoral dissertation, Ohio University, 1964). Dissertation Abstracts, 25:2329-2330, No. 4, 1964/65.

equation in a manner similar to Bassham[42] and predicted the post-test score. The difference between the actual score and the predicted score was the "level of achievement." No significant difference was found between "level of achievement" and (1) degree held, (2) years of training, (3) recency of training, and (4) teachers' qualifications.

In 1965, Moore[43] pre-tested and post-tested the students in 10 fourth grade classes and 11 sixth grade classes with the SRA Arithmetic Series Grades 4-6. He tested the 21 teachers with Glennon's test. He found no significant relationship between teacher understanding and pupil gain in achievement in arithmetic.

In 1965, Shim[44] used a different approach. He looked at the cumulative effect of 87 teachers who taught 214 students while they were in attendance in grades one through five. He measured student achievement (in arithmetic, language, and reading) with the California Achievement Test Form W Elementary. The four teacher variables were: (1) college grade-point average, (2) degree, (3) certificate, and (4) experience. He then dichotomized each of these variables and checked all possible hypotheses which relate teacher variables to student achievement. He concluded:

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[42]Bassham, loc. cit.

[43]R. E. Moore, "The Mathematical Understanding of the Elementary School Teacher as Related to Pupil Achievement in Intermediate-Grade Arithmetic" (unpublished Doctoral dissertation, Stanford University, 1965). Dissertation Abstracts, 26:213-214, No. 1, 1965/66.

[44]Chung-Phing Shim, "A Study of the Cumulative Effect of Four Teacher Characteristics on the Achievement of Elementary School Pupils," The Journal of Educational Research, 59:33-34, September, 1965.

> There is no significant difference in pupil achievement to support the idea that an elementary teacher has to be a superior student in college, to have a degree, to be fully certified, or to have many years of experience in order to be successful as far as measurable pupil achievement is concerned.[45]

In 1965, Railsback[46] investigated the reliability and validity coefficients of two different instruments which were developed to measure certain facets of teacher effectiveness. A team of raters evaluated 25 elementary teachers on both instruments. The Iowa Test of Basic Skills was administered to the students at the end of the year. A weak nonsignificant relationship was found between ranking of effectiveness and pupil achievements. In view of the many studies which have shown no significant relationship when ratings of teachers by observers as to their effectiveness is compared to achievement or gain, it is not surprising that Railsback found no significant relationship.

In 1967, Hurst[47] failed to find a relationship between the number of hours of college mathematics possessed by a teacher and student gain scores derived from administration of The Metropolitan Achievement Test. His population was 55 third grade teachers and their students. To obtain student gain he used the same procedure as Hall;[48] that is, he used the school's records and obtained successive

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[45] Ibid., p. 34.

[46] C. E. Railsback, "A Comparison of the Reliability and Validity of Two Types of Criterion Measures for Evaluation of Instruction" (unpublished Doctoral dissertation, University of Iowa, 1965). Dissertation Abstracts, 26:5829, No. 6, 1965/66.

[47] D. Hurst, "The Relationship Between Certain Teacher-Related Variables and Student Achievement in Third Grade Arithmetic" (unpublished Doctoral dissertation, Oklahoma State University, 1967).

[48] Hall, loc. cit.

September scores on the test. Again, one must critically question the summer effects on such a procedure and the lack of specifity of the tests used.

Rouse,[49] also reporting in 1967, used a technique similar to Shim.[50] He found no relationship between arithmetic achievement of fourth grade students and the total mathematics preparation of the teachers responsible for their instruction from kindergarten through grade six. The arithmetic achievement of the students was measured by the California Achievement Tests. The total mathematics preparation of the teachers was the mathematics courses they had completed in high school, college, and in-service.

In 1970, Cox[51] tested third graders and sixth graders with the SRA Achievement Series, Arithmetic, 1964. She classified the teachers of these students as high, average, or low as determined by their scores on Dr. Leroy Callahan's[52] Test of Mathematical Understanding. She found no significant results when she made the comparison between teacher knowledge as measured by Callahan's test and pupil mean-gain as measured by the pre-test post-test procedure. She did report that for the sixth graders there was a nonsignificant positive

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[49] Rouse, loc. cit.

[50] Shim, loc. cit.

[51] L. S. Cox, "A Study of Pupil Achievement in Mathematics and Teacher Competence in Mathematics" (unpublished Doctoral dissertation, University of Kansas, 1970). Dissertation Abstracts, 31:2767-A, 2768-A, No. 6, 1970/71.

[52] L. G. Callahan, "A Study of Knowledge Possessed by Elementary School Teachers, In-Service and In-Training, of Cultural, Psychological, and Mathematical Foundations of the Elementary School Program" (unpublished Doctoral dissertation, Syracuse University, 1966).

relationship. Pupils of high classified teachers made larger gains than did pupils of average classified teachers. Further, students of average classified teachers made larger gains than did students of low classified teachers. She did not report any relationships, significant or not significant, for the third graders.

In summarizing the elementary school studies, it seems they use the same techniques over and over and get the same results. Even in 1970 Cox[53] did not change the procedure. If we are to find any relationships we will have to change some ways of obtaining information and some ways of analyzing the information. Houston and DeVault[54] moved toward a more appropriate approach when they developed specific instruments to measure the desired goals.

When using the pre-test post-test idea, care must be taken to assure that gain is being measured. Using results obtained in successive Septembers leaves the results open to several serious questions. Even testing in September and May can be questioned because most of September, October, and November are commonly spent in review. If certain understanding is possessed by the student, then the teacher's understanding or lack of understanding will have little effect on the student during this period. The true effect of the teacher might better be obtained by pre-testing and post-testing around well identified and controlled blocks of novel material.

Attitudinal Reviews

A great amount of time and effort has gone into efforts to construct attitude scales that give an individual's attitude to a

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[53]Cox, loc. cit.

[54]Houston and DeVault, loc. cit.

particular matter. Much more time and effort will be spent with similar results; that is, results which must be suspect because of the instruments used.

In 1956, Poffenburger and Norton[55] asked 16 college seniors to complete a questionnaire relative to their attitudes toward mathematics and when they developed these attitudes. After reviewing the results they concluded:

> 1. Parents determine initial attitudes of their children toward arithmetic.
>
> 2. Parents' expectations of their children's performance and the encouragement they give in regard to the study of arithmetic affect children's achievement.
>
> 3. Arithmetic and mathematics teachers can have strong positive or negative effects upon students' attitudes and achievement.[56]

Also working with college students, Purcell,[57] in 1964, studied the effect of certain factors on attitude change toward elementary mathematics in a group of prospective teachers. The Dutton Arithmetic Attitude Scale[58] was used to determine student attitude. Purcell reported a significant correlation between attitude in elementary mathematics and understanding of elementary mathematics, but reported a nonsignificant correlation when comparing improved understanding

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[55] T. N. Poffenburger and D. D. Norton, "Factors Determining Attitudes Toward Arithmetic and Mathematics," The Arithmetic Teacher, 3:113-116, April, 1956.

[56] Ibid., p. 116

[57] W. J. Purcell, "Some Factors Affecting Attitudes of Prospective Teachers Toward Elementary Mathematics" (unpublished Doctoral dissertation, Teachers College, Columbia University, 1964).

[58] W. H. Dutton, "Measuring Attitudes Toward Arithmetic," Elementary School Journal, 55:24-31, September, 1954.

with favorable attitude change. He also reported a nonsignificant correlation when comparing favorable attitude change with a high grade in course work. This study seems to indicate that if a student understands the material he has a favorable attitude, but a favorable change in attitude does not assure increased understanding or better grades.

O'Donnell,[59] in 1958, examined 109 college seniors in elementary education with the California Achievement Test, Mathematics Section, Grades 9 to 14, Form W, to determine their arithmetic proficiency. He also administered H. H. Remmers'[60] attitude scale, Scale to Measure Attitudes Toward Any School Subject, to find student attitude toward arithmetic. He found that attitude toward arithmetic showed only a low nonsignificant correlation with arithmetical achievement and arithmetical problem solving behavior. White,[61] in 1962, disagreed with O'Donnell after evaluating 92 college students, enrolled in a methods course for elementary school arithmetic, with the Dutton Attitude Scale and Test D: Basic Arithmetic Skills of the Iowa Every-Pupil Tests of Basic Skills, Advanced Battery. She reported significant positive changes occurred in students' attitudes toward arithmetic, and

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[59] J. R. O'Donnell, "Levels of Arithmetic Achievement and Attitude Toward Arithmetic and Problem Solving by Prospective Elementary Teachers" (unpublished Doctoral dissertation, Pennsylvania State University, 1958). Dissertation Abstracts, 19:1300, No. 6, 1958/59.

[60] H. H. Remmers, N. L. Gage, and J. F. Rummel, A Practical Introduction to Measurement and Evaluation, (New York: Harper and Brothers, 1960), pp. 285-342.

[61] M. J. White, "A Study of the Change of Achievement and Attitude Toward Arithmetic by Prospective Elementary School Teachers Under Conditions of Television" (unpublished Doctoral dissertation, Wayne State University, 1963). Dissertation Abstracts, 25:2302-2303, No. 4, 1964/65.

significant gains were made in vocabulary and fundamental knowledge, computations, and total arithmetic achievement.

The three studies, Purcell,[62] O'Donnell,[63] and White,[64] are indicative of the studies which attempt to relate attitude and achievement in arithmetic as it relates to pre-service training of teachers. Since these studies and other similar studies are contradictory, one must question the procedures. It is not clear what is being measured when standardized tests and attitude scales are being used. It is evident that different procedures are necessary to determine whether or not there is a relationship between attitude and achievement in arithmetic.

Studies at the secondary level are no more conclusive. Goldberg et al.[65] found that attitudes of junior high school students toward arithmetic were not correlated to their gain in achievement in arithmetic. This is in agreement with O'Donnell[66] at the secondary level. Goldberg et al., wrote:

> Why the students who showed the greatest gains in achievement did not also show more positive attitudes toward mathematics is a question which cannot be answered from the data.[67]

Peskins[68] working with seventh graders compared teacher attitude in arithmetic with student attitude in arithmetic and with student achievement in arithmetic. Out of 24 possible correlations between teacher attitude and student attitude or student achievement, 15 were

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[62] Purcell, loc. cit.

[63] O'Donnell, loc. cit.

[64] White, loc. cit.

[65] Goldberg et al., loc. cit.

[66] O'Donnell, loc. cit.

[67] Goldberg et al., op. cit., p. 261.

[68] Peskins, loc. cit.

negative and 2 of these correlations were significant. These results indicate that a teacher's attitude toward mathematics might play an inverse role in affecting the students' attitude or achievement. A significant positive relationship did exist between the teachers' understanding of arithmetic and pupils' attitude toward arithmetic. Garner[69] found no significant relationship between teacher attitude toward algebra and student achievement in algebra.

McCradle,[70] in 1959, reported, in what appears to be one of the most comprehensive and well designed studies to date, some relationships between teacher attitude and student achievement in first year algebra. His population was 29 teachers and 1642 students. He used the Minnesota Teacher Attitude Inventory to measure the teachers' attitude toward teaching. He then classified the teachers as high, middle, or low, depending upon the results of the attitude inventory. The students were evaluated in three areas: (1) quantitative thinking, (2) functional competence in mathematics, and (3) algebra achievement. McCradle found the students in classes of the high teacher group had significantly larger gains in quantitative thinking and functional competence in mathematics than did the students with teachers in the middle group or the low group. Further, the attitude of the teacher was not significantly related to pupil scores

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[69] Garner, loc. cit.

[70] J. H. McCradle, "An Investigation of the Relationship Between Pupil Achievement in First Year Algebra and Some Teacher Characteristics" (unpublished Doctoral dissertation, University of Minnesota, 1959). Dissertation Abstracts, 20:165, No. 1, 1959/60.

on the algebra achievement measure. Taylor,[71] though, could find no significant correlations between teacher attitude to pupils and high school science growth.

It seems that the evidence is again inconclusive. The study by McCradle[72] would seem to indicate that those teachers with a more positive attitude toward teaching do develop some characteristics in students, quantitative thinking and functional competence in mathematics, that other teachers do not develop. Again, more research is necessary to draw strong conclusions.

Two studies seem relevant at the elementary level. Smail[73] also used the Minnesota Teacher Attitude Inventory to measure teacher attitude toward teaching. He found a significant relationship between the attitude of the teacher toward teaching and pupil mean-gain in arithmetic. His population was fourth, fifth, and sixth graders. These results somewhat support the findings of McCradle.[74] Bassham, Murphy, and Murphy[75] compared student attitude to student achievement. They measured student attitude using Dutton's scale. They separated the students into two groups, over-achievers and under-achievers. They made this grouping on the basis of results from Kuhlman-Anderson Intelligence Tests and The Iowa Tests of Basic Skills (Reading Comprehension). They reported a significant relationship between attitude and classification as over-achiever and under-achiever. They further reported, though:

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[71]Taylor, loc. cit.

[72]McCradle, loc. cit.

[73]Smail, loc. cit.

[74]McCradle, loc. cit.

[75]H. Bassham, M. Murphy, and K. Murphy, "Attitude and Achievement in Arithmetic," The Arithmetic Teacher, 11:66-72, February, 1964.

> The wide variability in weighted achievement at both extremes of the distribution of attitude scale scores would indicate that prediction of achievement on the basis of attitude score for individuals would be hazardous.[76]

These studies seem to indicate that teacher attitude and/or student attitude toward arithmetic might have some relationship to student learning of arithmetic. By using wide-range attitude scales, most studies possibly lost the results necessary for significance. It might be that narrowing the scope of the attitude measured could lead to significant results.

Justification of the Study

As indicated in the above review, most of the studies did not test teacher knowledge or teacher attitude directly. They used principals' ratings, number of college courses taken, and other indirect measures. It might be expected that the findings of these studies would be less reliable than from studies in which these variables are directly measured. Heil et al. agreed, concluding: "Observers ratings, per se, are next to worthless as a criterion of teacher effectiveness."[77] Medley and Mitzel,[78] after reviewing conclusions from previous research involving supervisory ratings, came to a similar conclusion.

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[76] Ibid., p. 71.

[77] Heil et al., op. cit., p. 66.

[78] D. M. Medley and H. E. Mitzel, "Some Behavioral Correlates of Teacher Effectiveness," Journal of Educational Psychology, 50:239-246, December, 1959.

Five studies at the elementary level directly tested teacher knowledge with a paper and pencil test. Heil et al.[79] made no effort to determine teachers' understanding of arithmetic. They determined teacher knowledge by administering the Teacher Education Examination, which is a very general examination. Smail,[80] Bassham,[81] and Moore[82] determined teacher understanding by administering Glennon's Test of Basic Mathematical Understandings[83] constructed in 1948. This test was designed to determine the understandings basic to computational processes taught in grades one through six at that time. Cox[84] administered Callahan's Test of Mathematical Understanding[85] for her criterion of teacher understanding. This test looks at many aspects of arithmetic and it is not clear what it is designed to measure, but it is not designed to measure only understanding in arithmetic. In view of recent developments in elementary school mathematics, it seems that teaching at the elementary level now involves more advanced understandings than those basic to the computational processes. It must therefore be concluded that the failure of these studies to report significant results may be attributable to the inprecision of the measures rather than to a lack of any underlying relationship. An attempt to detect any such underlying relationship must now result from an effort to measure precisely those teacher understandings

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[79] Heil et al., loc. cit.

[80] Smail, loc. cit.

[81] Bassham, loc. cit.

[82] Moore, loc. cit.

[83] Glennon, loc. cit.

[84] Cox, loc. cit.

[85] Callahan, loc. cit.

related to contemporary mathematics and to the syllabus of a specific elementary program.

For the same reasons mentioned above, it is now necessary to refine the measures of student growth. It might be worthwhile to partition student growth into three parts: (1) computation, (2) problem solving, and (3) understanding. One might expect considerable variation in the relationships between specific teacher variables and each of these parts of student growth in arithmetic. This might be especially true when relating teacher understanding to student understanding in arithmetic. None of the five studies mentioned above measured student growth in understanding.

All five of these studies used nationally normed standardized tests to determine student gain. It is probable that these tests did not adequately evaluate the goals of a given syllabus. More useful data might be obtained if the tests used to determine student growth were designed for the goals of the syllabus for the grades involved.

The review of the literature indicates that present information regarding the relationship between teacher variables, especially understanding in arithmetic, and student achievement and/or growth is inconclusive. Only one study, Bassham,[86] at the elementary level found a significant relationship and that relationship held for only above average students. The literature suggests that this inconclusiveness may be partially the result of insufficient identification

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[86] Bassham, loc. cit.

and measurement of the variables which are likely to be significant. In 1945, Barr[87] summarized:

> The success of the teacher depends no small part upon the extent that what she has to offer fits into the expectancy of pupils, parents, and school officials in the community in which she works. These individual determiners of teaching efficiency need further study.[88]

A teacher variable which deserves some consideration in today's world is the attitude of the teacher toward contemporary mathematics. This variable has not been compared with student gains in any study found by this researcher. It might be that this attitude has a relationship to the learning of contemporary mathematics.

Weaver and Gibb,[89] after reviewing the literature to 1964, concluded:

> Investigations such as these, however, leave unanswered the question concerning "cause and effect." Existing evidence is consistent with the hypothesis that teacher change begets pupil change of a like kind in mathematics. Nevertheless, one must look to the future for research designed specifically to test this hypothesis.[90]

The demands for more research in the area of teacher variables versus student gains seem to be great. They not only come from the reviewers of the literature, but from Departments of Mathematics who would like to adjust their programs to meet the needs of pre-service students, and from practicing teachers who would like to take in-service courses to better educate their students.

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[87] A. S. Barr, "Impressions, Trends, and Future Research," Journal of Experimental Education, 14:200-206, December, 1945.

[88] Ibid., p. 206.

[89] F. J. Weaver and G. E. Gibb, "Mathematics in the Elementary School," Review of Educational Research, 34:273-285, June, 1964.

[90] Ibid., p. 282.

The Study

Because of the problems, needs, and justification discussed above, this study will attempt, in a more sophisticated way than appears to have been attempted to date, to establish relationship between selected teacher variables and student growth in arithmetic. The teacher variables will be: understanding of arithmetic by direct testing, attitude toward contemporary mathematics, college courses taken in mathematics, how long since the last of these mathematics courses was taken, college courses taken in methods of teaching mathematics, how long ago was the last of these methods courses taken, number of quarter hours of professional education courses, number of years of teaching experience, number of years in present district, and principal's rating. Student growth in arithmetic will be partitioned into three parts: (1) computation, (2) problem solving, and (3) understanding. The reason for including teacher variables different from teacher understanding and teacher attitude is that some investigators have reported significance when using some of these variables. Further, other researchers have reported that a composite of these variables have a significant effect on student learning.

Hypotheses

The following null hypotheses will be checked:

H1. There is no significant relationship between selected teacher variables and student growth in computation.

H2. There is no significant relationship between selected teacher variables and student growth in problem solving.

H3. There is no significant relationship between selected teacher variables and student growth in understanding.

H4. There is no significant relationship between selected teacher variables and student growth in achievement.

Since no specific a priori hypotheses have been selected from among the huge number of possible interaction effects, any observations made of such interactions will be considered suggestions for further research.

Chapter 3

DESIGN OF THE STUDY

Definitions

There is considerable variation in the literature regarding definitions of terms used in mathematics. Although not all researchers would agree, this study will adopt the following definitions:

1. Elementary grades: kindergarten through sixth grade.

2. Secondary school: seventh grade through senior year in high school.

3. Computation: that part of arithmetic dealing with the algorithms of the real numbers.

4. Problem solving: that part of arithmetic dealing with worded problems and the establishment of equations which lead to correct solutions.

5. Understanding: that part of arithmetic dealing with the algebraic principles, the patterns, the fundamental properties of the real numbers, and the notational agreements accompanying them.

6. Growth: The difference between post-test and pre-test.

7. Achievement: growth in problem solving, computational skills, and understanding.

The Subjects

In an effort to overcome some of the design problems discussed in Chapters 1 and 2, the Spokane, Washington, and Bremerton, Washington,

school districts were chosen as the areas in which to carry out the study. They were chosen because:

1. They both used the Laidlaw Mathematics Series.[1]

2. They had very similar programs in elementary school mathematics.

3. Tests could be constructed which measure the material of the text and the programs.

4. Spokane is a metropolitan area of 200,000 people and is a transportation center in the eastern part of the state, while Bremerton is a city of 30,000 and is an industrial area in the western part of the state. A reasonable cross-section of the state's population was possible by using both cities.

5. The teacher population was large enough so that a random sample of the 400 teachers would ensure valid statistical treatment of the data.

6. Neither district groups students homogeneously. They are assigned to teachers on a random basis.

7. The teachers were the regular fourth, fifth, and sixth grade classroom teachers, all of whom met the state's certification requirements.

8. The students were fourth, fifth, and sixth grade students from the schools in the two districts.

Construction of Tests

Again, to overcome the stated deficiencies of the studies reported in Chapter 2, this researcher constructed tests designed to

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[1] B. H. Gundlach et al., Arithmetic (River Forest, Illinois: Laidlaw Brothers Publishers, 1964).

measure the material of the Laidlaw series and the understanding of students and teachers. Eleven different tests were constructed:

1. A test of teacher understanding.
2. An inventory to measure teacher attitude toward contemporary as opposed to traditional mathematics.
3. Nine student tests of arithmetic:
   (a) Three problem solving tests, one for each grade.
   (b) Three computation tests, one for each grade.
   (c) Three understanding tests, one for each grade.

Construction of a Test of Teacher Understanding

Because of the nonexistence of a test for practicing teachers that attempts to measure all areas of understanding, this researcher constructed a test for use in this study.

A set of 114 items was collected. Sixty-three of these items had been used by other researchers to measure understanding. Fifty-one of the items were constructed by this researcher. They were chosen and designed to measure understanding the teacher should possess so that she can teach the mathematical understanding stressed in the Laidlaw series, the series used in the two districts. All items were multiple choice. Some items had three choices, some had four choices, and some had five choices.

The 114 items were separated into three subtests. Each subtest was administered to 58 student teachers at the University of British Columbia. These subjects were students in the last month of a mathematics course for elementary teachers. An item analysis was performed and 16 items were removed. Many other items were rewritten as were many of the possible multiple choice responses.

The remaining 98 items were divided into two subtests. Each subtest had 55 items because 12 items were used in both subtests. These 12 items--four from each of the original subtests--were those which had the largest point biserial correlations with their tests. This procedure was used to attempt to prevent the point biserial correlation from being controlled by some set of items in one of the subtests.

Each of the two subtests was administered to 75 students at the University of British Columbia. These were students in the last month of a mathematics course for elementary teachers. An item analysis was performed and six items were removed from the test.

Because the subjects used for the first two administrations were students completing a mathematics course, it was assumed that their understanding of mathematics would be greater than practicing teachers. Therefore, careful consideration was given those items which seemed too difficult or too easy for these subjects.

The remaining 92 items were divided into two subtests with 12 items duplicated. Each of these subtests was given to 80 students in summer school at Eastern Washington State College, Cheney, Washington. The majority of these subjects, 68, were practicing teachers. Forty-one of the 68 were teachers of the fourth, fifth, or sixth grades. The item analysis eliminated 22 items leaving 70 items on the test.

Before these items were again used, each item was rewritten so that it had five possible answer choices, and one of the choices was "none of these" or its equivalent. These 70 items were then administered to 164 practicing elementary teachers attending summer school at Eastern Washington State College. Eighty-nine of these subjects were fourth, fifth, or sixth grade teachers.

None of the items were removed because they were too difficult or too easy. All the items were answered correctly by at least 23 percent of the subjects and none of the items were answered correctly by more than 81 percent of the subjects. Further item analysis indicated that 10 items were not excellent discriminators when the 50 high scorers were compared with the 50 low scorers. The difference was 15 percent or less when the correct percentage of the top 50 on a given item was compared with the correct percentage of the low 50 on the same item. Hence, these 10 items were removed from the test.

To estimate the minimum possible reliability of the test, the Kuder-Richardson 20 reliability coefficient[2] was calculated. The 60 remaining items had a Kuder-Richardson 20 reliability coefficient of .79. The data for this calculation was from the 164 summer school students identified above.

Because of the validation procedures and the high Kuder-Richardson 20 reliability coefficient, these 60 items were used as the Test of Teacher Understanding for this study. A copy of the instrument is Appendix A.

Construction of An Inventory to Measure Teacher Attitude Toward Contemporary Mathematics Opposed to Traditional Mathematics

Since 1957 the use of 'new,' 'modern,' or 'contemporary' mathematics has been on the increase throughout the continent. Even though these are in general use, it is questionable whether or not the majority of teachers have a positive attitude toward contemporary mathematics curricula.

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[2] G. F. Kuder and M. W. Richardson, "The Theory of the Estimation of Test Reliability," Psychometrika, 2:151-160, September, 1937.

In a review of the literature concerning teacher attitudes, no inventory was found that attempted to measure teachers' attitude toward contemporary as opposed to more traditional mathematics curricula. This experimenter, therefore, constructed such an inventory for use with elementary teachers and elementary education majors.

An instrument constructed by Rice,[3] in 1964, evaluated attitude toward modern mathematics as well as attitude toward mathematics. Because of the dual purpose of this instrument it was not deemed appropriate for the purposes desired. The correlation between this inventory and Rice's inventory is .75. This was computed from the results of 46 elementary majors at the University of British Columbia.

A list of factors which seem to reflect the differences between traditional mathematics and contemporary mathematics at the elementary level was generated as a result of a questionnaire circulated among a group of five authorities in the field. These factors were:

1. Teachers' general and/or overall reaction toward contemporary mathematics.

2. Teachers' opinions of computational speed and/or computational ability in mathematics.

3. Teachers' opinions of the place and/or the value of new topics in mathematics, e.g., set theory, other bases.

4. Teachers' opinions of student needs in mathematics and/or student reactions to mathematics.

5. Teachers' opinions of the place and/or the value of the principles of arithmetic in mathematics.

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[3] J. M. Rice, "A Study of Attitudes of Elementary Teachers Toward Modern Mathematics Programs" (unpublished Doctoral dissertation, Oklahoma State University, 1964).

6. Teachers' opinions of the methods of teaching arithmetic.

A list of 49 items was constructed with these factors as the guide. Each item was a statement which was followed by two choices from which the subject was to choose his response. The choices represent preference for modern mathematics curricula or preference for traditional mathematics curricula.

The first form (Appendix B) of the attitude inventory was administered to a summer school class of 18 students at the University of British Columbia. Most of these students were practicing elementary teachers. Their scores ranged from 28 to 44 with a mean of 35.39, a median of 34.5, and a standard deviation of 4.55. This class was also given the opportunity to comment on statements which they found ambiguous or misleading. An analysis of these results and comments led to the removal of seven items--3, 7, 11, 32, 33, 36, 39--and the rewriting of sixteen items.

The first form of this inventory was also administered to nine students at the master's level who were taking a course in mathematics education. These students were instructed to mark the choice for each statement that in their opinion indicated the stronger attitude toward contemporary mathematics. If at least eight of the nine students agreed on a response, that response was assumed to show the more positive attitude. These results and the comments of these students resulted in the rewriting of eleven items and the removal of two items--18 and 49.

To estimate the minimum possible reliability of the inventory, the Kuder-Richardson 20 reliability coefficient was calculated. This first form had a Kuder-Richardson 20 reliability coefficient of .64.

The second form of the inventory (Appendix B), containing 40 items, was administered to 18 different summer school students, also at the University of British Columbia. Most of these students were practicing elementary teachers. Their scores ranged from 17 to 36 with a mean of 28.17, a median of 30, and a standard deviation of 5.03. The Kuder-Richardson 20 reliability coefficient for the second form was .73. The analysis of these results led to the removal of seven items--3, 4, 5, 13, 15, 22, 28--and the rewriting of eight items.

The third form of the inventory (Appendix B), containing 33 items, was administered to a class of 33 different summer school students at the University of British Columbia. Most of these students were practicing elementary teachers. The scores for these students ranged from 9 to 30 with a mean of 21.76, a median of 22, and a standard deviation of 5.16. The Kuder-Richardson 20 reliability coefficient for the third form was .78. Item analysis of these results led to the removal of eight items--4, 7, 12, 16, 22, 23, 27, 29. None of the remaining items were rewritten.

At the end of the third form of the inventory the teachers were asked to rate their attitude toward modern mathematics on a scale from 1 to 11. Their ratings ranged from 1 to 11 with a mean of 7.48, a median of 8, and a standard deviation of 2.24. The correlation between their scores on the inventory and their opinions was .68.

The fourth form of the attitude inventory (Appendix B), containing 25 items, was administered to 137 summer school students at the University of British Columbia. Most of these students were practicing elementary teachers. These 137 students had scores which ranged from 6 to 24 with a mean of 17.39, a median of 18, and a standard

deviation of 3.68. The Kuder-Richardson 20 reliability coefficient for this fourth form was .67.

As part of the item analysis of the 137 results, the point biserial correlation[4] coefficient of each item with the whole test was calculated. An examination of Table 1 indicates that three of the items--5, 8, 19--did not have significant correlation coefficients.

A factor analysis of the 137 results on the fourth form was performed by the computing center at the University of British Columbia. The program used was the factor analysis sample program from the IBM 360 scientific subroutine package[5] and the factor scores program from Cooley and Lohnes' Multivariate Procedures for the Behavioral Sciences.[6] It also uses the varimax procedure for analytical orthogonal rotation. It should be noted that Glass and Taylor[7] point out that this program gives only approximate factor scores.

Because of the conclusions by John B. Carrol,[8] tetrachoric correlation coefficients were used for the factor analysis instead of Pearsonian coefficients. To verify some of the problems mentioned by Carrol, a special run using 23 of the 25 items and Pearsonian correlation coefficients was made. The results gave eight factors which accounted

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[4]H. E. Garrett, Statistics in Psychology and Education, (New York: David McKay Company, Inc., 1958).

[5]IBM System/360 Scientific Subroutine Package. Programmer's Guide (360A-CM-03X).

[6]W. W. Cooley and P. R. Lohnes, Multivariate Procedures for the Behavioral Sciences, (New York: John Wiley and Sons, 1962).

[7]G. V. Glass and P. A. Taylor, "Factor Analytic Methodology," Review of Educational Research, 36:566-587, December, 1966.

[8]J. B. Carrol, "The Nature of the Data or How to Choose a Correlation Coefficient," Psychometrika, 26:347-372, December, 1961.

for 60 percent of the variance. The same data using tetrachoric correlation coefficients gave nine factors but accounted for 81 percent of the variance.

Table 1

Point Biserial Correlation

Item	Coefficient[a]
1	.38
2	.28
3	.27
4	.37
5	.20
6	.38
7	.37
8	.15
9	.35
10	.28
11	.30
12	.40
13	.52
14	.47
15	.30
16	.43
17	.33
18	.47
19	.01
20	.29
21	.45
22	.44
23	.38
24	.37
25	.28

[a].22 for significance at the .01 level.

The results of the factor analysis using all 25 items showed 10 factors which accounted for 81.5 percent of the variance. The rotated matrix was examined in an effort to determine which items contributed to these factors. It was assumed that a correlation of

.20 or larger between an item and a factor would identify those items that compose most of the factor. Table 2 shows the items which compose each factor.

Because of the lack of a significant point biserial correlation coefficient, the three items 5, 8, and 19 were removed and the teachers' scores were recalculated. The 137 teachers' scores ranged from 5 to 22 with a mean of 16, a median of 16, and a standard deviation of 3.59. The Kuder-Richardson 20 reliability coefficient for these 22 items was .71. It must be noted that a coefficient recalculated on original data in this fashion may be spuriously high.

Table 2

Items Which Have Correlation Coefficients of .20 or Greater with a Factor

Factors	Items
1	4, 7, 12, 13, 15, 18,[a] 20, 21,[a] 22,[a] 23,[a] 25
2	3,[a] 4, 6, 11, 14, 16[a] 17,[a] 23, 24[a]
3	1,[a] 11, 13, 15, 16, 22, 25[a]
4	1, 6,[a] 7, 9,[a] 13, 14,[a] 16, 18, 21
5	4,[a] 5,[a] 7, 9, 20, 21, 25
6	2,[a] 7,[a] 12, 13, 15, 16, 17
7	3, 4, 7, 10,[a] 13, 14, 15,[a] 20
8	1, 3, 4, 6, 7, 9, 11,[a] 12, 14, 21
9	3, 4, 8,[a] 13, 20[a]
10	6, 7, 12, 19,[a] 25

[a]A correlation greater than .5.

An examination of Table 3 indicates that all items had significant point biserial correlations with the whole test. Table 4 shows the results of the factor analysis on the 22 items. These eight factors accounted for 77.8 percent of the variance and were combinations of the original six factors which seemed to reflect the differences between traditional mathematics and modern mathematics at the elementary level.

Table 3

Point Biserial Correlation[a]

Item	Coefficient[b]
1	.36
2	.30
3	.29
4	.37
6	.35
7	.37
9	.34
10	.31
11	.30
12	.45
13	.55
14	.48
15	.29
16	.43
17	.35
18	.48
20	.33
21	.44
22	.46
23	.40
24	.38
25	.28

[a]22 items; 5, 8, and 19 removed.

[b].22 for significance at the .01 level.

The data from the third form of the test were then reanalyzed using just these 22 items. The correlation coefficient between the

teachers' scores on these 22 items and their opinion of their attitude toward contemporary mathematics was .79.

Because of the validation procedures and the results of the analyses on the 22 items, it was decided to use these 22 items as the inventory to determine teacher's attitude toward contemporary as opposed to traditional mathematics. A copy of the instrument is in Appendix B.

Table 4

Items Which Have Correlation Coefficients of .20 or Greater with a Factor[a]

Factors	Items
1	4, 7, 12, 13, 15, 18,[b] 22,[b] 23,[b] 25
2	3,[b] 4, 6, 11, 14, 16,[b] 17,[b] 23, 24[b]
3	2,[b] 7,[b] 12, 13, 15, 16, 17
4	3, 4, 7, 10,[b] 14, 15,[b] 16
5	1,[b] 7, 13, 15, 16, 22, 25[b]
6	1, 3, 4,[b] 6, 9, 11,[b] 12, 14
7	1, 6,[b] 7, 9,[b] 13, 14,[b] 16, 21
8	4, 7, 10, 13,[b] 18, 20[b]

[a] 22 items; 5, 8, and 19 removed.

[b] A correlation greater than .5.

Construction of Student Tests of Arithmetic

Three tests were constructed for each of the three grades: fourth, fifth, and sixth. These tests were tests of understanding, problem solving, and computation.

To obtain items for the fourth grade computation test, hereafter denoted C4, the fourth grade textbook was scrutinized and compared with the third grade textbook. All forms of computation new to the fourth grade were identified. A set of 40 items were constructed representative of these forms of computation. Similar procedures were used to obtain items for the fifth grade computation test and the sixth grade computation test, hereafter known as C5 and C6. Forty-four items were constructed for C5 and 54 items were constructed for C6.

To obtain items for the fourth, fifth, and sixth grade problem solving test, the textbooks for these grades were scrutinized and compared with textbooks from the previous grades. Problem solving procedures new to each of these grades were identified. Tests of 20 items each, one test for each grade, were constructed. The items were representative of the problem solving procedures new to each grade. These tests will hereafter be known as P4, P5, and P6.

To obtain items for the three tests of understanding, the textbooks of grades four, five, and six were scrutinized and the understandings were identified. One hundred thirteen items were constructed which were representative of these understandings. These items were randomly divided into two subtests. Form A contained 57 items and form B contained 56 items.

These 113 items were evaluated on a scale of one to seven by nine members of the Mathematics Education Department at the University of British Columbia. A score of one indicated no value as a measure of understanding while a score of seven indicated a high value as a measure of understanding. Any item which did not have a summed score of 27 or higher was removed from the test. One hundred one items remained with 50 in form A and 51 in form B.

The understanding items were multiple choice, while the computation and problem solving items called for constructed responses.

Since these tests were first administered in November, it was assumed that the fourth graders would approximate fourth graders at the beginning of the year, and the fifth graders would approximate fourth graders at the end of the year. The percentage of fourth graders who had an item correct was compared with the percentage of fifth graders who had the same item correct. This comparison gave an indication as to which items were fourth grade items in this text series. A similar comparison was made between the fifth and sixth graders to determine those items which seem to be fifth grade items in this text series. A similar comparison was made between the sixth and seventh graders to determine those items which seem to be learned in the sixth grade in this text series.

All administrations of these tests were in school districts which used the Laidlaw series.

Test C4 with 40 items was administered to 17 fourth graders and 16 fifth graders. The check to determine which items are fourth grade items and the item analysis led to the removal of 14 items. Test C4 was also administered to 50 different fourth and fifth graders to determine possible multiple choice distractors. The 26 items, as multiple choice items, were used for the second giving of C4. These items were administered to 26 fourth graders and 24 fifth graders. Item analysis eliminated one item leaving test C4 with 25 items. The reliability of this test and all other tests is given in Table 6 on page 61.

Test C5 was administered to 14 fifth graders and 16 sixth graders. The check to determine whether or not the items are fifth grade items and the item analysis led to the removal of 18 items. Test C5 was also administered to 53 different fifth and sixth graders to determine possible multiple choice distractors. The remaining 26 items, as multiple choice questions, were used for the second giving of C5. This form was administered to 13 fifth graders and 25 sixth graders. Item analysis eliminated one item leaving test C5 with 25 items.

Test C6 with 54 items was administered to 15 sixth graders and 12 seventh graders. The check to determine whether or not the items are sixth grade items and the item analysis removed 16 items. The 54 items were also given to 52 other sixth and seventh graders to determine possible multiple choice distractors. The remaining 38 items, as multiple choice questions, were used for the second administration. This form was administered to eight sixth graders and 32 seventh graders. Item analysis eliminated eight items leaving 30 items in test C6.

Test P4 with 20 items was administered to 13 fourth graders and 10 fifth graders. The check to determine whether or not these problems were fourth grade items and the item analysis eliminated five items. The 20 items were also administered to 50 different fourth and fifth graders to determine possible multiple choice distractors. The remaining 15 items, as multiple choice questions, were used for the second administration. It was administered to 26 fourth graders and 24 fifth graders. Item analysis did not eliminate any items leaving 15 items in test P4.

Test P5 with 20 items was administered to 13 fifth graders and 14 sixth graders. The check to determine whether or not these problems

were fifth grade items and the item analysis caused the removal of five items. The 20 items were also administered to 53 different fifth and sixth graders to determine possible multiple choice distractors. The remaining 15 items, as multiple choice questions, were used for the second administration. It was first separated into two subtests. One subtest of eight items was administered to 20 fifth graders and 24 sixth graders. The other subtest of seven items was administered to 18 fifth graders and 23 sixth graders. Item analysis did not eliminate any items leaving 15 items in test P5.

Test P6 with 20 items was administered to 13 sixth graders and 10 seventh graders. The check to determine whether or not these problems were sixth grade items and the item analysis did not remove any of the items. The 20 items were also administered to 52 different sixth and seventh graders to determine possible multiple choice distractors. The 20 items, as multiple choice questions, were used for the second administration. They were first separated into two subtests. One subtest of 10 items was administered to 20 sixth graders and 31 seventh graders. The other subtest of 10 items was administered to 20 sixth graders and 32 seventh graders. Item analysis led to the removal of four items leaving 16 items in test P6.

Form A of the student test of understanding was administered to 16 fourth graders, 37 fifth graders, 22 sixth graders, and 30 seventh graders. Form B of the student test of understanding was administered to 33 fourth graders, 25 fifth graders, 33 sixth graders, and 28 seventh graders. After a check to determine grade level and the item analysis, a 65 item fourth grade test of understanding, an 89 item fifth grade test of understanding, and an 88 item sixth grade

test of understanding were constructed. These tests will hereafter be referred to as test U4, test U5, and test U6. There were items which were in all three tests of understanding.

The second administration of test U4 was to 22 fourth graders and 20 fifth graders. Item analysis eliminated 15 items leaving 50 items in test U4. The second administration of half of test U5 was to 20 fifth graders and 24 sixth graders, and the other half to 18 fifth graders and 23 sixth graders. The item analysis removed 25 items leaving test U5 with 55 items. For the second administration, test U6 was divided into two subtests of 44 items each. One subtest was administered to 20 sixth graders and 31 seventh graders. The other subtest was administered to 20 sixth graders and 30 seventh graders. Item analysis removed 24 items leaving 64 items in test U6. This completed the second administration of each of the nine tests.

The third administration had two purposes: to permit an additional analysis of the items and to calculate test-retest reliability coefficients. To facilitate administration, tests U4, P4, and C4 were combined as one test booklet, T4. Tests U5, P5, and C5 were combined as one test booklet, T5. Tests U6, P6, and C6 were combined as one test booklet, T6.

The third administration was before the Christmas vacation. It was again administered after the Christmas vacation. About one month elapsed between the two administrations. Table 5 shows the number of students participating in each administration.

Because of the differences in scores and number of items on each part of the test, a standardized score was computed for each student on the reduced set of items and the test-retest reliability coefficients

Table 5

Number of Students Involved for Third Administration

	T4		T5		T6	
Students	4th	5th	5th	6th	6th	7th
Involved	180	210	215	180	252	114
Answer sheets returned before Christmas	169	193	201	172	237	114
Answer sheets returned after Christmas[a]	165	196	203	170	238	--
Taking all the tests on both givings	131	159	160	151	197	--
Taking Test U before Christmas	163	186	196	171	228	105
Taking Test U after Christmas	159	192	196	169	233	--
Taking Test U on both givings	144	170	176	160	210	--
Taking Test P before Christmas	163	183	196	167	230	110
Taking Test P after Christmas	160	193	201	166	230	--
Taking Test P on both givings	145	167	181	155	211	--
Taking Test C before Christmas	156	184	192	166	230	111
Taking Test C after Christmas	162	195	196	166	228	--
Taking Test C on both givings	141	170	173	154	208	--

[a]The seventh graders did not participate after Christmas.

were computed. Table 6 shows the reliability of each of the tests and the reliability of test booklets T4, T5, and T6.

Table 6

Test-Retest Reliability Coefficients for Each Test and T4, T5, and T6

Test	Grade	U	P	C	T
T4	4th	.7677	.5695	.5763	.7966
	5th	.8679	.8017	.8191	.9280
	4th & 5th	.8581	.7864	.8176	.9215
T5	5th	.7417	.4485	.3265	.6664
	6th	.8073	.6661	.8145	.8870
	5th & 6th	.8089	.6466	.8004	.8795
T6	6th	.8574	.5461	.7365	.8350

To further ensure that the tests actually measure student growth at the given grade, the mean scores for the intended grade levels and following grade levels were calculated for each of the nine tests. Table 7 shows the means at the intended grade level and significantly higher means (at the 1 percent level) at the following grade level for each of the nine tests.

Item analysis reduced the number of items on each test as follows:

Test U4 removed 8 items leaving 42 items.

Test P4 removed 2 items leaving 13 items.

Test C4 removed 1 item leaving 24 items.

Test U5 removed 12 items leaving 43 items.

Test P5 removed 2 items leaving 13 items.

Test C5 removed 2 items leaving 23 items.

Test U6 removed 20 items leaving 44 items.

Test P6 removed 0 items leaving 16 items.

Test C6 removed 0 items leaving 30 items.

These reduced forms of the tests were then used as the student tests for this study. Copies of these tests are in Appendices C, D, and E.

Table 7

Mean Student Score on Each Test

Test	4th	5th	6th	7th
U4	15.29	21.69	--	--
P4	2.95	5.40	--	--
C4	6.22	11.67	--	--
U5	--	14.93	19.93	--
P5	--	3.19	5.27	--
C5	--	4.39	9.56	--
U6	--	--	19.90	24.23
P6	--	--	4.37	6.73
C6	--	--	9.41	14.33

Plan of The Study

During the spring of 1968, the appropriate administrators of Spokane, Washington, and Bremerton, Washington, granted permission to have the study performed in their school districts. The principals of both districts agreed to cooperate. It was mutually decided to carry out the study during the 1968-1969 school year.

To maintain the anonymity of teachers, each school was numbered and each teacher within the school was numbered. A six digit numeral was given to each student. The first two digits represented the school, the middle two digits represented the teacher, and the last two digits represented the students. The researcher did not know the names of the participating teachers and had no way of relating them to the data collected. The distribution of material was handled internally by each school district.

Ninety-nine teachers, 33 at each grade level, were randomly selected to participate in the study. Table 8 shows the number of teachers completing all aspects of the study. Table 9 shows the number of students completing the study.

Table 8

Number of Teachers Participating

Grade	4	5	6	Total
Spokane	14	14	12	40
Bremerton	7	9	5	21
Total	21	23	17	61

Table 9

Number of Students Participating

Grade	4	5	6	Total
Spokane	321	365	403	1089
Bremerton	164	218	140	522
Total	485	583	543	1611

In September, 1968, the principals were given the details of the study. In October, 1968, the principals administered the Teacher Test of Understanding and the attitude inventory, Attitude Toward Contemporary Mathematics.

Because some researchers reported a possible effect on student learning from a composite of teacher variables, it was decided to obtain information on other teacher variables:

1. Number of quarter hours taken in college mathematics.
2. Number of quarter hours of new mathematics.
3. How long since the last of these mathematics courses was taken.
4. Number of quarter hours of mathematics method courses.
5. How long since the last of these method courses was taken.
6. Number of quarter hours of professional education courses.
7. Number of years of teaching experience.
8. Number of years in present district.
9. Principal's rating of teachers.

The principal obtained all of this information, except his rating of the teacher, by having each teacher complete a questionnaire (Appendix F). This information was also collected in October, 1968. On the first of May, 1969, the principals were asked to rate their teachers (Appendix G). This rating was concerned with the teacher's ability to teach mathematics using a contemporary approach.

Statistical Procedure

The relationships between teacher variables and student growth were compared by multiple linear regression. The analysis was performed at the computer center of the University of British Columbia using the

"Botward" version of linear regression analysis. This version was originally presented by Robert A. Bottenberg and Joe H. Ward.[9]

Multiple linear regression produces R^2, the percentage of variance in the specified criterion that is accounted for by the specified predictor variables. The specified criteria in this study, the dependent variables, are:

1. Student growth in understanding.
2. Student growth in problem solving.
3. Student growth in computation.
4. Student achievement.

The specified predictor variables, the independent variables, are:

1. The raw score on the Teacher Test of Understanding.
2. The raw score on the attitude inventory, Attitude Toward Contemporary Mathematics.
3. The categorization of the quarter hours of college mathematics completed by each teacher such that:

   0 represents 0 quarter hours.

   1 represents 1 to 7 quarter hours.

   2 represents 7 to 13 quarter hours.

   3 represents 13 to 19 quarter hours.

   4 represents 19 or more quarter hours.

4. The categorization of the quarter hours of 'new' mathematics completed by each teacher such that:

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[9] R. A. Bottenberg and J. H. Ward, Applied Multiple Linear Regression, Clearinghouse for Federal Scientific and Technical Information, United States Department of Commerce, Technical Documentary Report PRL-TDR-63-6, March, 1963.

0 represents 0 quarter hours.

1 represents 1 to 7 quarter hours.

2 represents 7 to 13 quarter hours.

3 represents 13 to 19 quarter hours.

4 represents 19 or more quarter hours.

5. The categorization of the number of years since the last mathematics course was completed by each teacher such that:

0 represents the past year.

1 represents 1 to 2 years.

2 represents 2 to 5 years.

3 represents 5 to 10 years.

4 represents 10 or more years.

6. The categorization of the number of quarter hours of mathematics methods courses completed by each teacher such that:

0 represents 0 quarter hours.

1 represents 1 to 4 quarter hours.

2 represents 4 to 9 quarter hours.

3 represents 9 to 13 quarter hours.

4 represents 13 or more quarter hours.

7. The categorization of the number of years since the last mathematics methods course was completed by each teacher such that:

0 represents the past year.

1 represents 1 to 2 years ago.

2 represents 2 to 5 years ago.

3 represents 5 to 10 years ago.

4 represents 10 or more years ago.

8. The categorization of the number of quarter hours of professional education courses completed by each teacher such that:

0 represents 0 to 20 quarter hours.

1 represents 20 to 30 quarter hours.

2 represents 30 to 40 quarter hours.

3 represents 40 to 50 quarter hours.

4 represents 50 or more quarter hours.

9. The categorization of the number of years teaching experience by each teacher such that:

0 represents 0 years of experience.

1 represents 1 year of experience.

2 represents 2 years of experience

3 represents 3 or 4 years of experience.

4 represents 5 or 6 years of experience.

5 represents 7 to 10 years of experience.

6 represents 10 to 15 years of experience.

7 represents 15 to 20 years of experience.

8 represents 20 or more years of experience.

10. The categorization of the number of years teaching experience within the district by each teacher such that:

0 represents 0 years of experience.

1 represents 1 year of experience.

2 represents 2 years of experience.

3 represents 3 or 4 years of experience.

4 represents 5 or 6 years of experience.

5 represents 7 to 10 years of experience.

6 represents 10 to 15 years of experience.

7 represents 15 to 20 years of experience.

8 represents 20 or more years of experience.

11. The categorization of the knowledge of calculus by each teacher such that:

0 represents no calculus completed in college.

1 represents some calculus completed in college.

12. The rating of the teacher by his principal on a scale from one to seven, seven is superior, as to the ability of the teacher in general as a teacher.

13. The rating of the teacher by his principal on a scale from one to seven, seven is superior, as to the ability of the teacher as a mathematics teacher.

14. The rating of the teacher by his principal on a scale from one to seven, seven is superior, as to the amount of new mathematics used by the teacher.

The F ratio comparing R^2 from the full model to R^2 from the restricted model is then calculated. The probability that an F ratio this large or larger occuring by chance alone is then determined. If the probability value is less than 5 percent, then the hypothesis, no relationship between variables, will be rejected.

The following hypotheses will be checked at the 5 percent level of significance:

H1. There is no significant relationship between selected teacher variables and student growth in computation.

H2. There is no significant relationship between selected teacher variables and student growth in problem solving.

H3. There is no significant relationship between selected teacher variables and student growth in *understanding*.

H4. There is no significant relationship between selected teacher variables and student growth in achievement.

Many other models will also be checked for significance. This will enable the researcher to determine such things as, "Do years of teaching experience have an effect upon student growth in mathematics?" Since none of these models have been hypothesized, any indication of effect on learning must be considered subjects for future research.

Chapter 4

RESULTS

The first calculations to compare the teacher variables with the student variables were the Pearson Correlation Coefficients. Table 10 gives these correlations. A correlation coefficient of 0.250 or larger is required for significance at the 5 percent level. Only one correlation, the one comparing principal's rating of the teacher as a teacher and growth in computation, is significant. These results seem to indicate, contrary to many earlier studies, that the principal does have some idea who his 'best teachers' are when 'best teachers' are determined by student growth in arithmetic computation.

Multiple linear regression equations were then used to compare the teacher variables with each of the student variables. The data from Table 11 indicates the 14 teacher variables accounted for approximately 21 percent of the variance in the dependent variable, student growth in computation. When the related R^2, 0.2117, is compared with R^2 from the restricted model, 0, an F-ratio of 0.9712 is computed. This is clearly nonsignificant.

The data from Table 11 also indicates that the 14 teacher variables accounted for approximately 21 percent of the variance in the dependent variable, student growth in problem solving. The F-ratio of 0.9671 is again clearly nonsignificant. Further, the data from Table 11 indicates that the 14 teacher variables accounted for approximately 18 percent of the variance in the dependent variable,

Table 10

Correlation Coefficients Comparing Teacher Variables to Student Variables

Teacher Variables	Student Growth in Understanding	Student Growth in Problem Solving	Student Growth in Computation
Score on test of understanding	-0.0140	0.0672	-0.0764
Score on attitude inventory	0.0633	0.0855	0.0346
Quarter hours of college mathematics	0.0092	0.2232	-0.1841
Quarter hours of new mathematics	0.0280	0.0994	0.0988
Years since last mathematics course	-0.0495	-0.0087	-0.1136
Quarter hours of mathematics methods	-0.0687	0.1307	0.1564
Years since last methods course	0.0039	0.0752	-0.0178
Quarter hours of professional education	-0.0086	0.0488	-0.1091
Years of teaching experience	-0.1363	0.0532	-0.0425
Years of district teaching experience	-0.0164	0.0359	0.0456
Taken calculus	0.0317	0.1077	0.0207
Principal's rating as a teacher	0.1932	0.2155	0.3041
Principal's rating as a mathematics teacher	-0.0406	0.1021	0.2046
Principal's rating as the use of new mathematics	0.0708	0.2286	0.2416

student growth in understanding. Again, a nonsignificant F-ratio of 0.8099 was calculated. Lastly, Table 11 indicates the 14 teacher variables accounted for approximately 18 percent of the variance in the dependent variable, student achievement in arithmetic. Again, a nonsignificant F-ratio of 0.7733 was calculated.

Table 11

R^2 Results When the Teacher Variables Are Compared with Each of the Student Variables

Student Variables	R^2	F	Probability
Growth in computation	0.2117	0.9712	0.4924
Growth in problem solving	0.2110	0.9671	0.4961
Growth in understanding	0.1830	0.8099	0.6471
Growth in achievement	0.1762	0.7733	0.7413

Since none of the four a priori null hypotheses were rejected, it was decided to further examine the data by the technique commonly called 'data snooping' for any possible nonlinear relationship which might be used for further research.

In this technique, each variable is partitioned into a set of intervals, and each interval acts, at first, as an independent variable in a regression equation. If any apparent statistically significant relationship is indicated by this procedure, possible nonlinear hypotheses relating the sequential intervals are tested.

Only one possible relationship was found. When the principals' ratings as teachers, as mathematics teachers, and as teachers of modern mathematics are all dichotomized between four and five (on a

seven point scale), and the number of years since the last mathematics course was dichotomized between one and two years, then a regression equation using the above mentioned variables and the Teacher Test of Understanding, the Teacher Attitude Inventory, number of quarter hours of college mathematics, the number of quarter hours of 'new mathematics,' the quarter hours of methods courses, and the presence of a calculus course in the teacher's background as independent variables produced a possibly significant relationship ($p = 2.21$ percent) with the dependent variable, student problem solving.

Very little confidence may be placed in this result. In enough 'data snooping' a significant correlation is almost bound to turn up sooner or later. No single correlation in this equation was high enough to encourage further exploration.

Chapter 5

CONCLUSIONS

The review of the literature indicated most researchers who looked for teacher variables which might relate to teacher effectiveness did not measure teacher variables precisely. Further, most researchers neither partitioned nor measured directly student growth. They used standardized tests or administrative ratings to determine teacher effectiveness.

This researcher constructed two instruments to measure teacher knowledge and teacher attitude. One was a test of mathematical understanding and the other was an inventory to measure the attitude toward contemporary mathematics as distinct from traditional mathematics.

To determine teacher effectiveness, this researcher constructed student tests to measure directly the material learned by the students. Three tests were constructed for each grade level; an understanding test, a problem solving test, and a computation test.

Because other researchers reported on a variety of teacher variables, this researcher obtained information about 12 other commonly reported variables. These were quarter hours of college mathematics, quarter hours of new mathematics, quarter hours of mathematics methods, experience, and principal's ratings as he reviewed the teacher. The main purpose for obtaining information on these variables was to determine whether or not testing procedures such as those used in this study would yield significant relationship.

The population for this study was 61 fourth, fifth, and sixth grade classes and their 61 teachers. They were randomly selected from the Spokane, Washington, and Bremerton, Washington, public schools.

The correlation coefficients comparing the 14 teacher variables with the three student variables were checked for significance. Only one, that comparing principals' ratings of teachers as general teachers and student growth in computation, was significant. This indicates that if student growth in computation is carefully measured by specific pre-test post-test procedures, then the principal's rating is corr lated to the effectiveness of the teacher. This result is contrary to the findings reported in most earlier studies, but this is the first data based on tests designed to measure growth in computation at a specific level and for a specific text book and arithmetic program. Therefore, if teacher effectiveness is precisely measured, the principal's rating of the teacher seems to significantly correlate with teacher effectiveness.

Next, the following four null hypotheses were tested:

H1. There is no significant relationship between selected teacher variables and student growth in computation.

H2. There is no significant relationship between selected teacher variables and student growth in problem solving.

H3. There is no significant relationship between selected teacher variables and student growth in understanding.

H4. There is no significant relationship between selected teacher variables and student growth in achievement.

The results would seem to indicate that none of the 14 variables, when taken individually or as a group, would be acceptable predictors of student growth in any of the three areas--understanding, problem solving, and computation--of arithmetic.

In this study, every effort was made to eliminate the deficiencies of previous studies. Yet their results are, in general, confirmed. Even the very tolerant sanctions of 'data snooping' turned up nothing that was not revealed in the first table of correlations.

There remains the opinion of many college instructors who train future teachers that there is a relationship between teacher variables and teacher effectiveness. If this is in fact the case, it seems that different independent variables must be identified. It seems highly unlikely that success would reward any further exploration of those identified in this study.

It should be observed that the entire population of teachers for this study were professionally trained. It would be a gross over-generalization to suppose that these results support the hypotheses that this professional training did not influence subsequent behavior in teaching, or that professional training is unnecessary.

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Appendix A

UNDERSTANDING INVENTORY

Teacher Number_____

Following is a set of questions which you are to answer. Place your answer on the blank to the left of each question. You may guess if you wish. Answer as many as you can in the 45 minute time limit. You may begin.

____ 1. The numeral $\frac{3}{4}$ can also be thought of as

A) 3 x 4
B) 4 x 3
C) 3 ÷ 4
D) 4 ÷ 3
E) none of these

____ 2. If θ is a binary operation defined in S and if for all a, b in S, $a \theta b = b \theta a$, then θ is said to obey the:

A) associative law
B) commutative law
C) distributive law
D) identity property
E) none of these

____ 3. If a, b, m, n are whole numbers different from zero, then $\frac{a}{m} + \frac{b}{n} =$

A) $\frac{a + b}{m}$
B) $\frac{a + b}{n}$
C) $\frac{a + b}{m + n}$
D) $\frac{an + bm}{mn}$
E) none of these

____ 4. What is the best reason for placing the decimal point after the 3 instead of some other place?

$$\begin{array}{r} .52 \\ 1.42 \\ 1.32 \\ \hline 3.26 \end{array}$$

A) to keep the decimal points in a straight line
B) six decimal places in the problem divided by 3, the number of addends, equals 2
C) hundredths added to hundredths equals hundredths
D) because the answer must be larger than any addends

____ 5. Given a fractional number, if the denominator of the fractional number is decreased and the numerator is kept the same, then the new number is:

A) larger than the old number
B) smaller than the old number
C) approaching one
D) the same as the old number
E) unknown in relationship to the old number from the information given

____ 6. Look at b ÷ a where "a" and "b" are both whole numbers greater than one. How does the answer compare with "b"?

A) the answer is greater than b
B) the answer is smaller than b
C) can't tell until we see both whole numbers
D) can't tell until we see b
E) can't tell until the division is done

____ 7. The following statement shows a property of arithmetic
$4 \times (5 + 6) = (4 \times 5) + (4 \times 6)$
Which of the following shows the same property?

A) $2 \times (3 \times 4) = 4 \times (2 \times 3)$
B) $(6 \times 5) \times 7 = 6 \times (5 \times 7)$
C) $(5 \times 6) + (3 \times 4) = 8 \times 10$
D) $8 \times 6 = (8 \times 4) + (8 \times 2)$
E) none of these

____ 8. Look at the problem 439 x 450. How would the answer be changed if two zeros were placed to the right of 439 and the zero removed from 450? The answer would be:

A) the same as the old answer
B) one-tenth as large as the old answer
C) ten times larger than the old answer
D) one-hundredth as large as the old answer
E) none of these

____ 9. Part of this addition problem was accidentally erased from the chalkboard. Each "X" shows where a digit used to be. These digits were not necessarily the same. What digit belongs on the question mark?

A) 9
B) 7
C) 5
D) 2
E) none of these

```
 XXXX1
+   ?
------
XXXXXX
```

____10. Here is a sequence whose first term is 12. From any term in the sequence you can get the next term by adding 12: 12, 24, 36, 48, 60 . . . . . What is the 100th term of this sequence?

A) 6000
B) 1212
C) 1200
D) 112
E) none of these

____11. Which of the following would give the correct answer to 21 x 21?

A) the sum of 2 x 21 and 1 x 21
B) the sum of 20 x 21 and .1 x 21
C) the sum of 10 x 2.1 and 20 x 2.1
D) the sum of 1 x 2.1 and 20 x 2.1
E) none of these

____12. In the example $\begin{array}{r} 749 \\ \times\ 36 \end{array}$ you multiply by the 6, then by the 3. How do the two results (partial products) compare?

A) the second represents a number one-half as large as the first
B) the second represents a number twice as large as the first
C) the second represents a number five times as large as the first
D) the second represents a number ten times as large as the first
E) none of these

____13. Given a fractional number, if the numerator of the fractional number is decreased and the denominator is kept the same, then the new number is:

A) larger than the old number
B) smaller than the old number
C) approaching one
D) the same as the old number
E) unknown in relationship to the old number from the information given

____14. Given the example 368 x 24, then change 368 to 3680 and 24 to 2.4. The new answer would be:

A) the same as the answer for the original example
B) one-tenth as large as the answer for the original example
C) ten times larger than the answer for the original example
D) one hundred times larger than the answer for the original example
E) none of these

____15. For two sets M and N, the set of elements that are in both M and N is called:

A) the union of M and N
B) the intersection of M and N
C) the complement of M with respect to N
D) the cross product of M and N
E) none of these

____16. Which of the following will give the same answer as 13 x 23?

A) (10 x 20) + (3 x 3)
B) (10 x 20) + (10 x 3)
C) (13 x 20) + (3 x 3)
D) (13 x 20) + (13 x 3)
E) none of these

____17. The number, $\sqrt{5}$, is irrational. So also is:

A) $\sqrt{5} \times \sqrt{5}$
B) $\sqrt{5} + \sqrt{5}$
C) $\sqrt{5} - \sqrt{5}$
D) $\sqrt{5} \div \sqrt{5}$
E) none of these

____18. If * is an operation and it is replaced by +, then -, then x, then ÷, then (u * v) * w = u * (v * w) will be true exactly:

A) zero times when all four replacements are tried
B) one time when all four replacements are tried
C) two times when all four replacements are tried
D) three times when all four replacements are tried
E) none of these

____19. If a binary operation "*" is defined on any pair of real numbers "c" and "d" such that c * d = 2c + d, then 3 * 4 is equal to:

A) 11
B) 10
C) 12
D) 14
E) none of these

____20. If "r" is the multiplicative inverse of n, then

A) n + r = n
B) n x r = n
C) n + r = 0
D) n x r = 1
E) none of these

____21. Which of the following numbers is smaller than 2.047?

A) 2.111
B) 2.048
C) 2.050
D) 2.1
E) none of these

____22. When working 12 x .5 we get 6 as an answer. The best reason for the answer being smaller than 12 is

A) 12 is larger than .5
B) .5 is smaller than 12
C) we are finding how many halves in 12
D) we are finding half of 12
E) we are multiplying by a decimal

____23. Peter is asked to bring 13 bushels of potatoes from the barn to the house. He can carry 3 bushels in each trip. How many trips will Peter make?

A) 4 trips
B) $4\frac{1}{3}$ trips
C) 5 trips
D) can't be determined from the information given
E) none of these

____24. When we compare .60 and $\frac{5}{9}$ we find that .60 is

A) larger than $\frac{5}{9}$
B) smaller than $\frac{5}{9}$
C) the same size as $\frac{5}{9}$
D) unknown in size to $\frac{5}{9}$

____25. If every element of a set M is an element of a set N, then M is:

A) a subset of N
B) a proper subset of N
C) equivalent to N
D) an element of N
E) none of these

____26. When both the numerator and denominator of a fraction are divided by the same number, then the value of the new fraction is:

A) greater than the value of the old fraction
B) less than the value of the old fraction
C) the same as the value of the old fraction
D) unknown until the number used to divide is known
E) unknown until the fraction is known

____27. In a division problem the quotient is the same as the dividend when:

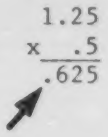
A) the divisor is less than one
B) the divisor is less than one but greater than zero
C) the divisor is a factor of the dividend
D) the divisor is greater than one
E) none of these

____28. Twenty-three acres of a 55 acre farm is used to raise corn. The part of the farm used to raise corn is slightly more than:

A) $\frac{3}{5}$
B) $\frac{2}{3}$
C) $\frac{2}{5}$
D) $\frac{1}{3}$
E) none of these

____29. What is the best reason for placing the decimal point before the 6 instead of some place else?

A) counting all decimal places you get three
B) the rule for multiplying decimals tells us to put it there
C) tenths times hundredths equals thousandths
D) since .5 equals .500, this keeps the decimal points in a straight line



____30. Look at b ÷ a where "a" and "b" are both proper fractions. How does the answer compare with "b"?

A) the answer is larger than b
B) the answer is smaller than b
C) can;t tell until I see the numbers
D) the answer is the same as b
E) can't tell until it is worked

____31. The best way to explain why moving the decimal point does not change the answer in the example

$.5\overline{)35.55}$ or $5.\overline{)355.5}$ is

A) when dividing by a decimal number you move the decimal in the divisor and the dividend the same number of places
B) there is no way to divide by a decimal without moving the decimal point
C) the rule for dividing decimal numbers tells us to move the decimal point the same number of places in the divisor and the dividend
D) moving the decimal point is the same as multiplying the numerator and denominator of a fraction by the same number
E) it is easier to divide by a whole number than a decimal number

____32. Look at u x v where "u" and "v" are both proper fractions. How does the answer compare with "u"?

A) the answer is larger than u
B) the answer is smaller than u
C) the answer is equal to u
D) can't tell until we see the fractions
E) can't tell until we do the arithmetic

____33. If a relation "R" defined in a set "S" has the property that for all a, b, c, and S, if a R b and b R c, then a R c, it is said to be:

A) commutative
B) reflexive
C) associative
D) transitive
E) none of these

____34. Part of this subtraction problem was accidentally erased from the chalkboard. Each "X" shows where a digit used to be. What digit belongs on the blank?

A) 0
B) 1
C) 6
D) 7
E) none of these

$$\begin{array}{r} XX_6 \\ -XX77 \\ \hline XX9X \end{array}$$

____35. A rational number expressed as a decimal fraction will <u>never</u> be:

A) infinite and have a repeating decimal expansion
B) infinite and have a nonrepeating decimal expansion
C) a terminating decimal expansion
D) none of these

____36. Considering each of the following as a separate problem, the problem in which the lowest common denominator will be one of the denominators is:

A) $\frac{1}{5} + \frac{3}{10} + \frac{4}{15}$
B) $\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$
C) $\frac{1}{3} + \frac{7}{12} + \frac{5}{6}$
D) $\frac{1}{8} + \frac{1}{2} + \frac{5}{6}$
E) none of these

____37. The value of the 4 in relation to the 2 in the number 4032 is:

A) 1000 times as large
B) 2 times as large
C) 500 times as large
D) 2000 times as large
E) none of these

____38. If θ is a binary operation defined in S and if for all a, b, c in S, $(a \theta b) \theta c = a \theta (b \theta c)$, then θ is said to obey the:

A) associative law
B) commutative law
C) distributive law
D) identity property
E) none of these

____39. When a natural number is divided by a proper fraction how does the answer compare with the natural number?

A) the answer is larger than the natural number
B) the answer is smaller than the natural number
C) the answer is equal to the natural number
D) can't tell until we see the natural number and the proper fraction
E) can't tell until we do the arithmetic

____40. $\frac{8}{12} \div 1$ equals which of the following?

A) $\frac{3}{6}$

B) $\frac{2}{3}$

C) $\frac{3}{4}$

D) $\frac{7}{11}$

E) none of these

____41. If r + s = t, then which of the following is also correct?

A) 4r + 4s = 8t
B) 4r + 4s = 4t
C) 4r + s = 5t
D) 4r + 4s = t x t x t x t
E) none of these

____42. When a natural number is multiplied by a proper fraction, how does the answer compare with the natural number?

A) the answer is greater than the natural number
B) the answer is smaller than the natural number
C) can't tell until we see the numbers
D) the answer is the same as the natural number
E) can't tell until it is worked

____43. If "p" is the additive inverse of "q", then:

A) p + q = 1
B) p + q = 0
C) p x q = 1
D) p + 0 = q
E) none of these

____44. If a, b, m, n are whole numbers different than zero, then $\frac{a}{m} \div \frac{b}{n} =$

A) $\frac{ab}{mn}$

B) $\frac{mn}{ab}$

C) $\frac{bm}{an}$

D) $\frac{an}{bm}$

E) none of these

____45. Which of the following also stands for 4275?

A) 42 hundreds + 75 tens
B) 427 tens + 5 ones
C) 4 thousands + 27 hundreds + 5 tens
D) 4 thousands + 2 hundreds + 75 tens
E) none of these

____46. Given the example $6.5\overline{)84.5}$, then change 6.5 to .65 and 84.5 to 845. The new answer would be:

A) the same as the answer for the original example
B) one-tenth as large as the answer for the original example
C) ten times larger than the answer for the original example
D) one hundred times larger than the answer for the original example
E) none of these

____47. 5.5 is equal to:

A) five and 50 tenths
B) five and 50 hundredths
C) five and 5 hundredths
D) five and 50 units
E) none of these

____48. 6 x 7 x 8 = (6 x 7) x 8 shows a property of arithmetic. Which of the following shows the same property?

A) 6 x 7 x 8 = 6 x (8 x 7)
B) 6 x 7 x 8 = 8 x (6 x 7)
C) 6 x 7 x 8 = (7 x 6) x 8
D) 6 x 7 x 8 = 6 x (7 x 8)
E) none of these

____49. If θ is a binary operation in the system of whole numbers W and if a θ 0 = 0 θ a = a for all "a" in W, then this is an example of the:

A) associative law
B) distributive law
C) commutative law
D) identity property
E) none of these

____50. If a + b = c, and "d" is any number, then which of the following is always true?

A) a + b + d = c - d
B) b + d = c - (a + d)
C) d x (a + b) = d + c
D) d x c = (d x a) + (d x b)
E) none of these

____51. If f + g > h, which statement is ALWAYS true for all real numbers f, g, and h?

A) $3f + 3g > 3h$

B) $f + \frac{5}{3}g > \frac{5}{3}h$

C) $\frac{2}{3}f + g < h$

D) $g > h$

E) none of these

____52. If the quotient when dividing 6.7 by .04 is the same as dividing "n" by 4.0, then "n" is:

A) 6700
B) 670.0
C) .67
D) .067
E) none of these

____53. Considering each of the following as separate problems, the problem in which the lowest common denominator will be the product of denominators is:

A) $\frac{1}{2} + \frac{1}{4} + \frac{1}{5}$

B) $\frac{1}{5} + \frac{5}{7} + \frac{1}{3}$

C) $\frac{5}{6} + \frac{7}{8} + \frac{3}{10}$

D) $\frac{1}{3} + \frac{1}{4} + \frac{1}{6}$

E) none of these

____54. If θ and ϕ are binary operations defined on a set S and if for all a, b, c in S, $a \theta (b \phi c) = (a \theta b) \phi (a \theta c)$, then we say

A) the associative law holds in S
B) the commutative law holds in S
C) the distributive law holds in S
D) the identity property holds in S
E) none of these

____55. Which fractional number is between 2 and 3?

A) $\frac{3}{2}$
B) $\frac{2}{3}$
C) $\frac{11}{5}$
D) $\frac{13}{4}$
E) none of these

____56. "n" in the problem .7 ÷ n = .007 is:

A) 1000
B) .01
C) 100
D) .001
E) none of these

____57. Given the problem $\begin{array}{r} 23.90 \\ \times\ 4.47 \end{array}$ how would the answer be changed if the zero were removed from 23.90? The new answer would be:

A) the same as the old answer
B) one-tenth as large as the old answer
C) ten times larger than the old answer
D) larger than the old answer because there would be fewer decimal places
E) unknown until we do the multiplication

____58. If you can add every number in a set of numbers to itself or to every other number in the set, and the sum is a number also in that set, then that set of numbers is closed under the operation of addition. Which of the following sets is NOT closed under the operation of addition?

A) 0, 1, 2, 3, 4, ...
B) 1, 2, 3, 4, 5, ...
C) 0, 2, 4, 6, 8, ...
D) 1, 3, 5, 7, 9, ...
E) none of these

____59. Part of this multiplication problem was accidentally erased from the chalkboard. Each "X" shows where a digit used to be. These digits were not necessarily the same. What digit belongs on the question mark?

A) 0
B) 2
C) 4
D) 5
E) 7

$$\begin{array}{r} XXX? \\ \times\ \ XX \\ \hline XXXX \\ XXX5 \\ \hline XXXX0 \end{array}$$

____60. An irrational number expressed as a decimal fraction will <u>always</u> be:

A) infinite and have a repeating decimal expansion
B) infinite and have a nonrepeating decimal expansion
C) a terminating decimal expansion
D) none of these

Appendix B

AN INVENTORY TO MEASURE TEACHER'S ATTITUDE
TOWARD CONTEMPORARY AS OPPOSED TO TRADITIONAL MATHEMATICS

First Form

Following is a multiple choice inventory.

Complete each statement so that your choice best reflect your beliefs, opinions, and practices.

If you do not know the meaning of some word in a statement, use "D" as your choice.

Place your answer on the blank to the left of each statement.

Thank you for your cooperation.

____ 1) New mathematics is

A) a success and here to stay
B) an educational fad which will pass on

____ 2) Students

A) dislike arithmetic because it is a dry subject
B) like arithmetic because it is full of new ideas

____ 3) In the teaching of arithmetic

A) we should stress that equal values may have different forms, in other words $3 = 2 + 1 = 2 + 1 + 1$.
B) it is not necessary to stress that equal values may have different forms

____ 4) There is considerable discussion as to whether fractions should be written as $\frac{2}{3}$ or as 2/3. This is an

A) unimportant distinction
B) important distinction

____ 5) When teaching multiplication by the number one it

A) should be emphasized as a special property
B) need not be emphasized

____ 6) When I teach arithmetic I do so because

A) I must teach it as part of the curriculum
B) I enjoy watching students learning arithmetic

____ 7) When learning arithmetic it is best to

A) discover those things we should learn
B) be told those things we should learn

____ 8) In a new mathematics program memorization of the arithmetic facts is

A) not as important as in the old programs
B) just as important as in the old programs

____ 9) When teaching addition and subtraction, it is best to teach them as

A) inverse operations
B) separate operations

____10) Students

A) dislike arithmetic because of the repetitious homework
B) like arithmetic because of the opportunity to think things out

____11) All operations of arithmetic are defined operations,

A) therefore it is not necessary to have understandings attached to them
B) but it is still important to have understandings attached to them

____12) Tables such as the multiplication tables

A) are not important in new mathematics because they stress memorization
B) are important in new mathematics because they assist in understanding number relationships

____13) Considering the world we live in, students should spend more time

A) studying mathematics
B) studying such things as art and music

____14) Non-positional numeration systems such as the Roman System are

A) unimportant in arithmetic today
B) important in arithmetic today

____15) When teaching arithmetic, the difference between number and numeral is

A) unimportant and should not be stressed
B) important and should be stressed

____16) In new mathematics checking an answer is

A) important because it helps lead to better understandings
B) unimportant because the child has the understandings before he does the work

____17) The associative property of addition is

A) necessary to understand addition of three or more numbers
B) unnecessary to understand addition of three or more numbers

____18) Tables such as the multiplication tables

A) are not important in new mathematics
B) are important in new mathematics

____19) When I teach arithmetic I

A) enjoy the teaching
B) dislike the teaching

____20) Most of the arithmetic taught is

A) of practical use
B) designed to build background for future study in mathematics

____21) The ability to calculate is

A) less necessary in the new mathematics
B) just as important as ever

____22) Work in bases different from ten

A) should be performed with most students
B) should not be performed with most students

____23) In new mathematics it is

A) just as important as before that students calculate rapidly
B) not as important as before that students calculate rapidly

____24) Student understanding of arithmetic is

A) less necessary today because of the many machines that do our calculating
B) more necessary today because of scientific advancements

____25) Words such as commutative, associative, and distributive

A) are important words in mathematics and by the end of the third grade, most children should know the words
B) represent important ideas in mathematics and by the end of the third grade, most children should understand these ideas--the words are not important

____26) High school algebra is

A) a prestige course
B) a course most students should and can take

____27) In arithmetic the teacher should teach the student to

A) do the work and understand it by practicing
B) understand what he is doing

____28) Set theory

A) fundamentals should be studied with arithmetic
B) is a separate branch of mathematics and, therefore, has little effect on learning arithmetic

____29) From my experiences I

A) like new mathematics best
B) like old mathematics best

____30) Computational shortcuts in arithmetic are

A) not as important as in the past
B) just as important as in the past

____31) New mathematics is better for

A) all students
B) college bound students only

____32) Understandings in arithmetic should be stressed with

A) the college capable student only--the slow learner has to memorize anyway
B) all students

____33) When solving story problems there

A) is one best way to do the problem and the teacher should stress this way
B) are many ways to do any problem and the teacher should accept any logically correct method

____34) New mathematics will train the student to

A) ask why
B) accept what he is told

____35) The number line

A) is for use in understanding algebra and need not be used before grade seven
B) can be used to help understand addition of whole numbers

____36) Arithmetic should be taught as a

A) set of rules for the students to follow
B) step by step process where one step builds upon the other

____37) When teaching division of whole numbers, it is best to use

A)
```
    234
56)13104
   112
   190
   168
    224
    224
```

B)
```
56)13104
 11200   200
  1904
  1680    30
   224
   224     4
           234
```

____38) For learning and understanding the multiplication algorithm, the understanding of the distributive property is

A) quite important
B) relatively unimportant

____39) New mathematics

A) shows the student the structure of arithmetic
B) has the student memorize certain definitions and then asks the student to use these memorized definitions

____40) In arithmetic zero is a

A) number as is one, two, three, etc.
B) placeholder

____41) When teaching the addition facts

A) the commutative property is presented so students know $a + b = b + a$
B) it is not necessary to stress the commutative property because the idea is simple enough for the students to realize $a + b = b + a$

____42) Realizing there is more than one way to subtract, it is best to

A) teach the student all ways
B) teach the student only one way so he will not become confused

____43) When introducing division of fractions, it is best to use

A) $\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$

B) $\frac{1}{4} \div \frac{1}{3} = \frac{3}{12} \div \frac{4}{12} = \frac{3 \div 4}{12 \div 12} = \frac{3 \div 4}{1} = 3 \div 4 = \frac{3}{4}$

____44) For learning and understanding the multiplication algorithm, the knowledge of place value is

A) quite important
B) relatively unimportant

____45) When introducing addition of two digit whole numbers, it is best to use

A)
$$\begin{array}{r} 13 \\ +\ 5 \\ \hline 18 \end{array}$$

B) $13 + 5 = (10 + 3) + 5 = 10 + (3 + 5) = 10 + 8 = 18$

____46) For learning and understanding the multiplication algorithm, the associative property of multiplication is

A) quite important
B) relatively unimportant

____47) Arithmetic

A) is seldom boring to the student
B) usually boring to the student

____48) When teaching multiplication and division, it is best to teach them as

A) inverse operations
B) separate operations

____49) For learning and understanding the multiplication algorithm

A) the distributive property is most important
B) knowledge of place value is most important

Second Form

Following is a multiple choice inventory.

Complete each statement so that your choice best reflects your beliefs, opinions, and practices.

If you do not know the meaning of some word in a statement, use "D" as your choice.

Place your answer on the blank to the left of each statement.

Thank you for your cooperation.

____ 1) New mathematics is

A) a success and here to stay
B) an educational fad which, as many have in the past, will pass on

____ 2) Students

A) dislike arithmetic because it is a dry subject
B) like arithmetic because it is full of new ideas

____ 3) There is considerable discussion as to whether fractions should be written as $\frac{2}{3}$ or as 2/3. This is an

A) unimportant distinction
B) important distinction

____ 4) When teaching multiplication by the number one it

A) should be emphasized as a special property
B) need not be emphasized

____ 5) When I teach arithmetic I do so because

A) I must teach it as part of the curriculum
B) I enjoy watching students learning arithmetic

____ 6) In a new mathematics program early rote memorization of the arithmetic facts is

A) not as important as in the old programs
B) just as important as in the old programs

____ 7) When teaching subtraction, it is better to teach it as

A) the inverse operation of addition
B) a separate operation

____ 8) Students

A) dislike arithmetic because of the repetitious homework
B) like arithmetic because of the opportunity to think things out

____ 9) Tables such as the multiplication tables

A) are not important in new mathematics because they stress memorization
B) are important in new mathematics because they assist in understanding number relationships

____10) Considering the world we live in, elementary students should spend more time studying

A) mathematics
B) such things as art and music

____11) Non-positional numeration systems such as the Roman System are

A) unimportant in new mathematics
B) important in new mathematics

____12) When teaching arithmetic, the difference between number and numeral is

A) unimportant and should not be stressed
B) important and should be stressed

____13) In new mathematics proving an answer correct is

A) as important as in old mathematics
B) unimportant

____14) The associative property of addition is

A) necessary to understand column addition
B) unnecessary to understand column addition

____15) When I teach arithmetic I

A) enjoy the teaching
B) dislike the teaching

____16) Most of the arithmetic taught by elementary teachers should be designed

A) for applications in practical life
B) to build background for future study in mathematics

____17) The ability to calculate with large numbers is

A) less necessary in new mathematics
B) just as important as ever

____18) Work in bases different from ten should

A) be performed with most students
B) not be performed with most students

____19) In light of the philosophy of new mathematics, calculating with great speed is

A) just as important as before
B) not as important as before

____20) Student understanding of arithmetic is

A) less necessary today because calculating machines are used to do the difficult calculations
B) more necessary today

____21) Words such as commutative, associative, and distributive

A) are important words in mathematics and by the end of the third grade, most children should know these words
B) represent important ideas in mathematics and by the end of the third grade, most children should understand these ideas--the words are not important

____22) Ninth grade algebra is a

A) prestige course
B) course most students should and can take

____23) In teaching arithmetic the teacher should teach so that the student

A) learns to do the work and then understands it as he practices or applies it
B) understands what he is doing

____24) Set theory

A) should be studied with the early introduction of arithmetic
B) is a separate branch of arithmetic and, therefore, should not be studied with the early introduction of arithmetic

____25) Because of my experiences, I like

A) new mathematics better
B) old mathematics better

____26) The teaching of computational shortcuts in arithmetic is

A) not as important in new mathematics as in old mathematics
B) just as important in new mathematics as in old mathematics

____27) New mathematics is better for

A) all students
B) college bound students only

____28) New mathematics will help train the students to

A) ask why things are happening in the world today
B) accept the things that are happening in the world today

____29) The number line is for use in understanding

A) negative numbers in algebra and mathematics beyond algebra
B) arithmetic as well as algebra and mathematics beyond algebra

____30) When teaching division of whole numbers, the algorithm I would use is

A)
```
   234
56)13104
  112
  190
  168
   224
   224
```

B)
```
56)13104
11200   200
 1904
 1680    30
  224
  224     4
          234
```

____31) For learning and understanding the multiplication algorithm, the understanding of the distributive property is

A) quite important
B) relatively unimportant

____32) In arithmetic zero is a

A) number as is one, two, three, etc.
B) placeholder

____33) When teaching the addition facts

A) the commutative idea should be stressed
B) it is not necessary to stress the commutative idea at such an elementary level

____34) Realizing there is more than one algorithm for subtraction, it is best to

A) teach the student several algorithms and let him choose the one he wishes to use
B) teach the student only one algorithm so he will not become confused

____35) When introducing division of fractions, the algorithm I would use is

A) $\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$

B) $\frac{1}{4} \div \frac{1}{3} = \frac{3}{12} \div \frac{4}{12} = \frac{3 \div 4}{12 \div 12} = \frac{3 \div 4}{1} = 3 \div 4 = \frac{3}{4}$

____36) For learning and understanding the multiplication algorithm, knowledge of place value is

A) quite important
B) relatively unimportant

____37) When introducing addition of two digit whole numbers, the algorithm I would use is

A)
$$\begin{array}{r} 13 \\ +\ 5 \\ \hline 18 \end{array}$$

B) $13 + 5 = (10 + 3) + 5 = 10 + (3 + 5) = 10 + 8 = 18$

____38) For learning and understanding the multiplication algorithm, the associative property of multiplication is

A) quite important
B) relatively unimportant

____39) Arithmetic

A) is seldom boring to the student
B) usually boring to the student

____40) When teaching division, it is better to teach it as

A) the inverse operation of multiplication
B) a separate operation

Third Form

The following is an inventory to determine certain teacher attitudes toward elementary school arithmetic. Each statement has two possible completions. Complete the statements so that your choices best reflect your beliefs, opinions, and practices. Place your answer on the blank to the left of each statement.

If you do not know the meaning of some word in a statement, use "D" as your choice.

Thank you for your cooperation.

____ 1) New mathematics is

A) a success and here to stay
B) an educational fad which, as many have in the past, will pass on

____ 2) Students

A) dislike arithmetic because it is a dry subject
B) like arithmetic because it is full of new ideas

____ 3) In a new mathematics program early rote memorization of the arithmetic facts is

A) not as important as in the old programs
B) just as important as in the old programs

____ 4) When teaching subtraction, it is better to teach it as

A) the inverse operation of addition
B) a separate operation

____ 5) Students

A) dislike arithmetic because of the repetitious homework
B) like arithmetic because of the opportunity to think things out

____ 6) Teaching multiplication by the number one

A) is a special property and should be emphasized
B) is easy and need not be emphasized

____ 7) Considering the world we live in, elementary students should spend more time studying

A) mathematics
B) such things as art and music

____ 8) Non-positional numeration systems such as the Roman System are

A) unimportant in new mathematics
B) important in new mathematics

____ 9) When teaching arithmetic, the difference between number and numeral is

A) unimportant and should not be stressed
B) important and should be stressed

____10) The associative property of addition is

A) necessary to understand column addition
B) unnecessary to understand column addition

____11) The majority of the arithmetic taught by elementary teachers should be designed

A) for applications in practical life
B) to build background for future study in mathematics

____12) The ability to calculate with large numbers is

A) less necessary in new mathematics
B) just as important as ever

____13) Work in bases different from ten should

A) be performed by most students
B) not be performed by most students

____14) In light of the philosophy of new mathematics, calculating with great speed is

A) just as important as before
B) not as important as before

____15) Student understanding of arithmetic is

A) less necessary today because calculating machines are used to do the difficult calculations
B) more necessary today

____16) Words such as commutative, associative, and distributive

A) are important words in mathematics and by the end of the second grade, most children should know these words
B) represent important ideas in mathematics and by the end of the second grade, most children should know these ideas--the words are not important

____17) Students

A) should always obtain understandings before skills
B) sometimes need skills before understanding

____18) Set theory

A) should be studied with the introduction of arithmetic in grade one
B) is a separate branch of arithmetic and, therefore, should not be studied with the early introduction of arithmetic

____19) Because of my experiences, I like

A) new mathematics better
B) old mathematics better

____20) The teaching of computational shortcuts in arithmetic is

A) not as important in new mathematics as in old mathematics
B) just as important in new mathematics as in old mathematics

____21) New mathematics is better for

A) most students
B) more able students

____22) The number line is for use in understanding

A) negative numbers in algebra and mathematics beyond algebra
B) grade one arithmetic as well as higher mathematics

____23) When teaching division of whole numbers, the algorithm I would use is

A)
$$\begin{array}{r} 234 \\ 56\overline{)13104} \\ \underline{112} \\ 190 \\ \underline{168} \\ 224 \\ \underline{224} \end{array}$$

B)
$$\begin{array}{rr} & \\ 56\overline{)13104} & \\ \underline{11200} & 200 \\ 1904 & \\ \underline{1680} & 30 \\ 224 & \\ \underline{224} & \underline{4} \\ & 234 \end{array}$$

____24) For learning and understanding the multiplication algorithm, the understanding of the distributive property is

A) quite important
B) relatively unimportant

____25) In arithmetic zero is a

A) number as is one, two, three, etc.
B) placeholder

____26) When teaching the addition facts

A) the commutative idea should be stressed
B) it is not necessary to stress the commutative idea at such an elementary level

____27) Realizing there is more than one algorithm for subtraction, it is best to

A) teach the student several algorithms and let him choose the one he wishes to use
B) teach the student only one algorithm so he will not become confused

____28) When introducing division of fractions, the algorithm I would use is

A) $\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$

B) $\frac{1}{4} \div \frac{1}{3} = \frac{3}{12} \div \frac{4}{12} = \frac{3 \div 4}{12 \div 12} = \frac{3 \div 4}{1} = 3 \div 4 = \frac{3}{4}$

____29) For learning and understanding the multiplication algorithm, knowledge of place value is

A) quite important
B) relatively unimportant

____30) When introducing addition of two digit whole numbers, the algorithm I would use is

A)
$$\begin{array}{r} 13 \\ +\ 5 \\ \hline 18 \end{array}$$

B) $13 + 5 = (10 + 3) + 5 = 10 + (3 + 5) = 10 + 8 = 18$

____31) For learning and understanding the multiplication algorithm, the associative property of multiplication is

A) quite important
B) relatively unimportant

____32) Arithmetic is usually

A) enjoyable to students
B) unenjoyable to students

____33) When teaching division, it is better to teach it as

A) the inverse operation of multiplication
B) a separate operation

How do you feel about new mathematics? Let 11 be highly favorable and 1 be highly unfavorable toward new mathematics. Give yourself a score from 1 to 11 depending upon your own opinion of yourself.

_____

Fourth Form

The following is an inventory to determine certain teacher attitudes toward elementary school arithmetic. Each statement has two possible completions. Complete the statements so that your choices best reflect your beliefs, opinions, and practices. Place your answer on the blank to the left of each statement.

If you do not know the meaning of some word in a statement, use "D" as your choice.

Thank you for your cooperation.

____ 1) New mathematics is

A) a success and here to stay
B) an educational fad which, as many have in the past, will pass on

____ 2) In a new mathematics program early rote memorization of the arithmetic facts is

A) not as important as in the old programs
B) just as important as in the old programs

____ 3) Students

A) dislike arithmetic because of the repetitious homework
B) like arithmetic because of the opportunity to think things out

____ 4) Teaching multiplication by the number one

A) is a special property and should be emphasized
B) is easy and need not be emphasized

____ 5) Non-positional numeration systems such as the Roman System are

A) unimportant in new mathematics
B) important in new mathematics

____ 6) When teaching arithmetic, the difference between number and numeral is

A) unimportant and should not be stressed
B) important and should be stressed

____ 7) The associative property of addition is

A) necessary to understand column addition
B) unnecessary to understand column addition

____ 8) The majority of the arithmetic taught by elementary teachers should be designed

A) for applications in practical life
B) to build background for future study in mathematics

____ 9) Work in bases different from ten should

A) be performed by most students
B) not be performed by most students

____10) In light of the philosophy of new mathematics, calculating with great speed is

A) just as important as before
B) not as important as before

____11) Student understanding of arithmetic is

A) less necessary today because calculating machines are used to do the difficult calculations
B) more necessary today

____12) Students

A) should always obtain understandings before skills
B) sometimes need skills before understanding

____13) Set theory

A) should be studied with the introduction of arithmetic in grade one
B) is a separate branch of arithmetic and, therfore, should not be studied with the early introduction of arithmetic

____14) Because of my experiences, I like

A) new mathematics better
B) old mathematics better

____15) The teaching of computational shortcuts in arithmetic is

A) not as important in new mathematics as in old mathematics
B) just as important in new mathematics as in old mathematics

____16) New mathematics is better for

A) most students
B) more able students

____17) Students

A) dislike arithmetic because it is a dry subject
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____18) For learning and understanding the multiplication algorithm, the understanding of the distributive property is

A) quite important
B) relatively unimportant

____19) In arithmetic zero is a

A) number as is one, two, three, etc.
B) placeholder

____20) When teaching the addition facts

A) the commutative idea should be stressed
B) it is not necessary to stress the commutative idea at such an elementary level

____21) When introducing division of fractions, the algorithm I would use is

A) $\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$

B) $\frac{1}{4} \div \frac{1}{3} = \frac{3}{12} \div \frac{4}{12} = \frac{3 \div 4}{12 \div 12} = \frac{3 \div 4}{1} = 3 \div 4 = \frac{3}{4}$

____22) When introducing addition of two digit whole numbers, the algorithm I would use is

A)
$$\begin{array}{r} 13 \\ +\ 5 \\ \hline 18 \end{array}$$

B) $13 + 5 = (10 + 3) + 5 = 10 + (3 + 5) = 10 + 8 = 18$

____23) For learning and understanding the multiplication algorithm, the associative property of multiplication is

A) quite important
B) relatively unimportant

____24) Arithmetic is usually

A) enjoyable to students
B) unenjoyable to students

____25) When teaching division, it is better to teach it as

A) the inverse operation of multiplication
B) a separate operation

Final Form

The following is an inventory to determine certain teacher attitudes toward elementary school arithmetic. Each statement has two possible completions. Complete the statements so that your choices best reflect your beliefs, opinions, and practices. Place your answer on the blank to the left of each statement.

If you do not know the meaning of some word in a statement, use "D" as your choice.

Thank you for your cooperation.

____ 1) New mathematics is

A) a success and here to stay
B) an educational fad which, as many have in the past, will pass on

____ 2) In a new mathematics program early rote memorization of the arithmetic facts is

A) not as important as in the old programs
B) just as important as in the old programs

____ 3) Students

A) dislike arithmetic because of the repetitious homework
B) like arithmetic because of the opportunity to think things out

____ 4) Teaching multiplication by the number one

A) is a special property and should be emphasized
B) is easy and need not be emphasized

____ 5) When teaching arithmetic, the difference between number and numeral is

A) unimportant and should not be stressed
B) important and should be stressed

____ 6) The associative property of addition is

A) necessary to understand column addition
B) unnecessary to understand column addition

____ 7) Work in bases different from ten should

A) be performed by most students
B) not be performed by most students

____ 8) In light of the philosophy of new mathematics, calculating with great speed is

A) just as important as before
B) not as important as before

____ 9) Student understanding of arithmetic is

A) less necessary today because calculating machines are used to do the difficult calculations
B) more necessary today

____10) Students

A) should always obtain understandings before skills
B) sometimes need skills before understanding

____11) Set theory

A) should be studied with the introduction of arithmetic in grade one
B) is a separate branch of arithmetic and, therefore, should not be studied with the early introduction of arithmetic

____12) Because of my experiences, I like

A) new mathematics better
B) old mathematics better

____13) The teaching of computational shortcuts in arithmetic is

A) not as important in new mathematics as in old mathematics
B) just as important in new mathematics as in old mathematics

____14) New mathematics is better for

A) most students
B) more able students

____15) Students

A) dislike arithmetic because it is a dry subject
B) like arithmetic because it is full of new ideas

____16) For learning and understanding the multiplication algorithm, the understanding of the distributive property is

A) quite important
B) relatively unimportant

____17) When teaching the addition facts

A) the commutative idea should be stressed
B) it is not necessary to stress the commutative idea at such an elementary level

____18) When introducing division of fractions, the algorithm I would use is

A) $\frac{1}{4} \div \frac{1}{3} = \frac{1}{4} \times \frac{3}{1} = \frac{3}{4}$

B) $\frac{1}{4} \div \frac{1}{3} = \frac{3}{12} \div \frac{4}{12} = \frac{3 \div 4}{12 \div 12} = \frac{3 \div 4}{1} = 3 \div 4 = \frac{3}{4}$

____19) When introducing addition of two digit whole numbers, the algorithm I would use is

A)
$$\begin{array}{r} 13 \\ +\ 5 \\ \hline 18 \end{array}$$

B) $13 + 5 = (10 + 3) + 5 = 10 + (3 + 5) = 10 + 8 = 18$

____20) For learning and understanding the multiplication algorithm, the associative property of multiplication is

A) quite important
B) relatively unimportant

____21) Arithmetic is usually

A) enjoyable to students
B) unenjoyable to students

____22) When teaching division, it is better to teach it as

A) the inverse operation of multiplication
B) a separate operation

Appendix C

ARITHMETIC INFORMATION, FOURTH GRADE

Student Number_____

Following are three different sets of arithmetic questions. These questions will tell us what arithmetic you already know and what arithmetic you will learn during this school year. Some of the questions will be easy for you and some of the questions will contain arithmetic you have not had. Each question has four or five possible answers. Choose the answer you believe is correct and mark it on the answer sheet as the first three examples have been done.

1) If $4 + n = 9$, then $n = ?$.

A) 4
B) 5
C) 6
D) 7
E) none of these

Notice that "B" has been marked on the answer sheet for example 1 because 5 is the correct answer.

2) If $4 \times p = 16$, then $p = ?$.

F) 2
G) 3
H) 4
I) 5
J) none of these

Notice that "H" has been marked on the answer sheet for example 2 because 4 is the correct answer.

3) If $24 \div 3 = n$, then $n = ?$.

A) 2
B) 3
C) 4
D) 5
E) none of these

Notice that "E" has been marked on the answer sheet for example 3 because the correct answer, 8, is not given as a choice.

Good heavy pencil marks have been made in the correct spaces. This will be necessary when you mark your answers.

Now, do examples 4 and 5.

4) If $9 - p = 4$, then $p = ?$.

F) 2
G) 3
H) 4
I) 5
J) none of these

5) If $(1 + 2) + n = 7$, then $n = ?$.

A) 2
B) 3
C) 4
D) 5
E none of these

You should have marked "I" as the correct answer for example 4. You should have marked "C" as the correct answer for example 5.

You are now ready to answer the questions as you have done the examples above.

PART 1

Following is a set of 42 questions which will tell us what arithmetic understandings you know. Do not use paper to solve any of these questions. You should solve each of them in your head and then mark your answer on the answer sheet. Do not write in this test booklet or on the answer sheet.

You will have 32 minutes to complete Part 1. Stop at the end of Part 1. Do not go on to Part 2 until you are instructed to do so.

At this time, ask any questions you might have.

---

1) What would be the next number in the following set? 3, 6, 9, ----.

A) 10
B) 11
C) 12
D) 13
E) none of these

2) In the problem at the right, the number 1284 is "best" explained as being

```
  214
x 365
 1070
1284
642
78110
```

F) 600 x 214
G) 60 x 214
H) 6 x 214
I) 6 x 214 without a zero added
J) 6 x 214 moved to the left one place

3) The inverse operation of subtraction is

A) addition
B) subtraction
C) multiplication
D) division
E) none of these

4) The exercise $8\overline{)24}$ means

F) 8 times 24
G) how many subsets of 24 in a set of 8
H) how many subsets of 8 in a set of 24
I) 8 : 24
J) none of these

5) In the example at the right, what is the best reason for placing 68 one place to the left?

```
  34
x 23
 102
 68
 782
```

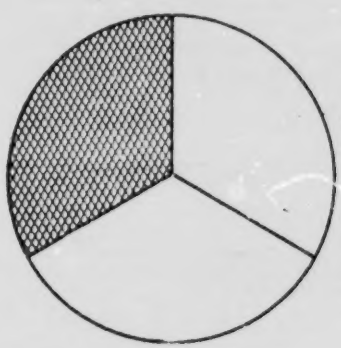
A) because the 8 must be under the 2
B) because this is the rule in multiplication
C) because the 68 is really 680
D) because 34 is a two digit number
E) because 23 is below 34 instead of above 34

6) The inverse operation of addition is

F) addition
G) subtraction
H) division
I) multiplication
J) none of these

7) The shaded part of the figure is what part of the figure?

A) $\frac{1}{2}$
B) $\frac{1}{3}$
C) $\frac{1}{4}$
D) $\frac{1}{5}$
E) none of these

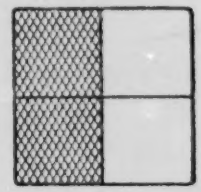


8) Which number is the largest?

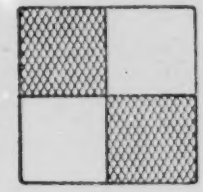
F) 7000
G) 6999
H) 7001
I) 7010
J) 7100

9) Look at the two squares. The shaded part of the square M is

M



N



A) less than the shaded part of square N
B) more than the shaded part of square N
C) equal to the shaded part of square N
D) cannot tell from the picture

10) In the number 7342, about how many thousands are there?

F) two thousands
G) four thousands
H) three thousands
I) seven thousands
J) eight thousands

11) In the expression 427 x 638 x 546, how will the answer be changed if it is worked as 546 x 638 x 427?

A) the answer will be less
B) the answer will be greater
C) the answer will be the same
D) can't tell until it is worked out

12) A mixed number such as $3\frac{3}{4}$ means

F) $3 \times \frac{3}{4}$

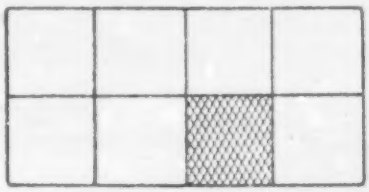
G) $3 + \frac{3}{4}$

H) $3 - \frac{3}{4}$

I) $3 \div \frac{3}{4}$

J) none of these

13) The shaded part of this figure is what fractional part of the figure?



A) $\frac{1}{2}$
B) $\frac{1}{3}$
C) $\frac{1}{4}$
D) $\frac{1}{5}$
E) none of these

14) The inverse operation of division is

F) addition
G) subtraction
H) multiplication
I) division
J) none of these

15) These statements are true: $4 + 6 = r$, $6 + 6 = s$, $4 + 4 = t$, $6 + 4 = u$. Which of the following is also true?

A) $r = s$
B) $r = t$
C) $r = u$
D) $s = u$
E) none of these

16) The inverse operation of multiplication is

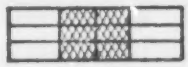
F) addition
G) subtraction
H) multiplication
I) division
J) none of these

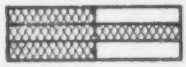
17) If $7 \times t = 0$, then "t" is always

A) zero
B) one
C) seven
D) ten
E) it is impossible to tell from the information given

18) Which shaded figure shows one-half of one-third?

F)

G) 

H) 

I)

J) none of these

19) As the denominator of a fractional number decreases and the numerator remains the same, the number

A) becomes larger
B) becomes smaller
C) remains the same
D) approaches one
E) can't tell from the information given

20) As the numerator of a fractional number decreases and the denominator remains the same, the number

F) becomes smaller
G) becomes larger
H) remains the same
I) gets close to one
J) can't tell from the information given

21) These statements are true: $a + b = d$, $c + b = e$, $c + c = f$. Which of the following is also true?

A) $a = b + d$
B) $c = b + e$
C) $d + e = f$
D) $e = b + c$
E) none of these

22) If $0 \times y = 0$, then "y" is always

F) zero
G) one
H) two
I) ten
J) any number you choose

23) Which of the following is another numeral for 526?

A) (5 x 10 x 10) + (2 x 10) + 6
B) (5 x 5 x 5) + (2 x 2) + 6
C) (5 x 100) + (2 x 100) + 6
D) (5 x 2 x 6 x 100) + (2 x 6 x 10) +6
E) none of these

24) A fraction such as $\frac{5}{7}$ means

F) choosing 7 parts after dividing an object into 5 parts
G) choosing 2 parts after dividing an object into 5 parts
H) choosing 5 parts after dividing an object into 7 parts
I) choosing 2 parts after dividing an object into 7 parts
J) none of these

25) If a x b = 0, then

A) "a" always equals "b"
B) "b" must be zero
C) "a" must be zero
D) either "a" or "b" must be zero
E) none of these

26) Which fraction is the smallest?

F) $\frac{1}{3}$
G) $\frac{2}{3}$
H) $\frac{1}{4}$
I) $\frac{3}{4}$
J) $\frac{1}{2}$

27) If r x s = r, then "s" is always

A) zero
B) one
C) "r"
D) ten
E) it is impossible to tell from the information given

28) Jimmy's bike has a speedometer which shows miles and tenths of miles. It looks like this now | 027 | 7 |. How far will he ride before the speedometer reads | 033 | 0 |?

F) more than 5 miles
G) less than 5 miles
H) exactly 5 miles
I) can't tell from the information given

29) If r + s = r, then "s" is always

A) zero
B) one
C) "r"
D) ten
E) it is impossible to tell from the information given

30) To subtract in the exercise $\begin{array}{r} 645 \\ -162 \end{array}$, we should

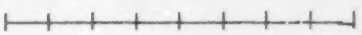
F) make the 5 ones smaller
G) make the 5 ones larger
H) make the 4 tens smaller
I) make the 4 tens larger
J) none of these

31) If u-v = u, then "v" is always

A) zero
B) one
C) "u"
D) ten
E) it is impossible to tell from the information given

32) In the exercise $7\overline{)364}$, the 5 really stands for

F) 5 ones
G) 5 tens
H) 5 hundreds
I) 5 tenths
J) none of these

33) This line segment is cut into 

A) sevenths
B) eighths
C) ninths
D) tenths
E) none of these

34) Four thousand three hundred seven is written

F) 4037
G) 0437
H) 4370
I) 40003007
J) none of these

35) Look at the numeral 85,626. The 6 on the left has a value how many times larger than the 6 on the right?

A) 100 times larger
B) 10 times larger
C) 1000 times larger
D) the same value
E) none of these

36) When multiplying in the problem $\begin{array}{r} 937 \\ \times\ 84 \\ \hline \end{array}$ we move the second partial product, which we get when we multiply by 8, one place to the left because

F) that is the rule in multiplying
G) the 8 means 8 tens
H) the answer must be larger than 937
I) the top number is a number larger than ten
J) none of these

37) Look at the problem p - q. If "q" is the identity number for this subtraction, then "q" is equal to

A) zero
B) one
C) ten
D) there isn't one
E) none of these

38) The number 4357 is about

F) 4 hundreds
G) 43 hundreds
H) 435 hundreds
I) 4357 hundreds

39) Round off 9766 to the nearest hundred.

A) 9770
B) 9700
C) 9800
D) 10,000
E) none of these

40) How would the sum in the problem $\begin{array}{r} 64 \\ +78 \\ \hline \end{array}$ be changed if 78 was placed above 64 instead of below it?

F) the new sum would be less than the old sum
G) the new sum would be the same as the old sum
H) the new sum would be greater than the old sum
I) it is unknown until it is worked both ways

41) Multiplying 6 and 9 is the same as

A) increasing 6 by 9
B) adding six-ninths
C) adding nine-sixths
D) adding six nines
E) none of these

42) How would the product in the problem $\begin{array}{r} 7608 \\ \times\ 47 \end{array}$ be changed if 47 was placed above 7608 instead of below it?

F) the new product would be the same as the old product
G) the new product would be greater than the old product
H) the new product would be less than the old product
I) it is impossible to multiply when the larger number is on the bottom and the smaller number on the top
J) it is unknown until it is worked both ways

END OF PART 1

DO NOT TURN THE PAGE

PART 2

Following is a set of 13 problems which will tell us what arithmetic problems you can work.

You may use paper to solve these problems. Put three clean sheets of paper on your desk to figure on. Do not write in this test booklet or on the answer sheet.

You will have 22 minutes to complete Part 2. Do not go on to Part 3 until you are instructed to do so.

At this time, ask any questions you might have.

---

1) Joe paid $8.68 for 7 baseballs. How much did each baseball cost?

A) $60.75
B) $ 1.25
C) $ 1.24
D) $ 1.09
E) none of these

---

2) Marilyn bought $\frac{1}{3}$ of a yard of ribbon. How many inches did she buy?

F) 24 inches
G) 12 inches
H) 15 inches
I) 36 inches
J) none of these

---

3) Joe could put 6 pictures on each page of his photo album. He had 77 pictures. How many full pages could he mount and how many pictures are left?

A) 72 pages and 5 pictures
B) 66 pages and 1 picture
C) 462 pages and 0 pictures
D) 12 pages and 5 pictures
E) none of these

---

4) John wishes to buy a bicycle which sells for $37.50. He now has $19.75. He will work to earn enough money to buy the bicycle. How much money must he earn?

F) $15.75
G) $17.75
H) $19.75
I) $57.25
J) none of these

---

5) Jane waited $\frac{1}{5}$ of an hour for her mother. How many minutes did she wait?

A) 10 minutes
B) 12 minutes
C) 15 minutes
D) 20 minutes
E) none of these

6) David bought a pair of pants for $8.95 and a shirt for $3.75. How much change did he receive from $20.00?

F) $ 7.30
G) $ 8.30
H) $12.70
I) $14.80
J) none of these

---

7) Peter bought 3 baseballs at $1.25 each, 2 bats at $1.75 each, and one glove at $5.95. How much did Peter spend?

A) $ 8.95
B) $ 9.95
C) $12.20
D) $13.20
E) none of these

---

8) The eight members of the Boys Club bought 12 bottles of pop at 8 cents for each bottle and 6 dozen cookies at 32 cents for each dozen. If the members share the cost equally, how much did each member pay?

F) $ 2.88
G) 40¢
H) 44¢
I) 24¢
J) none of these

---

9) Frank had $1.75 to buy school supplies. His father gave him $4.50. While shopping he spent $5.37. How much money did he have left?

A) $ 6.25
B) 98¢
C) 88¢
D) 78¢
E) none of these

---

10) Mrs. Smith needs 375 cookies. She has 22 packages, each containing 11 cookies. How many more cookies does she need?

F) 133 cookies
G) 342 cookies
H) 353 cookies
I) 242 cookies
J) none of these

---

11) Mrs. Johnson made 36 cookies on Monday, 45 cookies on Tuesday, and 27 cookies on Wednesday. By Saturday half of the cookies were gone. How many cookies did she have left?

A) 108 cookies
B) 54 cookies
C) 44 cookies
D) 40 cookies
E) none of these

---

12) Farmer Brown sold 355 pounds of hay in January, 267 pounds of hay in February, and 216 pounds of hay in March. On the average, how many pounds of hay did he sell each month?

F) 212 pounds
G) 266 pounds
H) $279\frac{1}{3}$ pounds
I) 838 pounds
J) none of these

13) Jane picked $2\frac{1}{2}$ dozen asters and $1\frac{1}{2}$ dozen roses. How many flowers did Jane pick?

A) 31 flowers
B) 3 dozen flowers
C) $3\frac{2}{4}$ dozen flowers
D) 4 dozen flowers
E) none of these

END OF PART 2

DO NOT TURN THE PAGE

PART 3

Following is a set of 24 problems which will tell us what arithmetic problems you can work.

You may use paper to solve these problems. Put three clean sheets of paper on your desk to figure on. Do not figure in this test booklet or on the answer sheet.

You will have 29 minutes to complete Part 3.

At this time, ask any questions you might have.

1)	2478 6002 6201 5743	A) 21,424 B) 20,424 C) 20,224 D) 20,244 E) none of these	2)	692 -457	F) 245 G) 235 H) 135 I) 145 J) none of these
3)	90,006 -36,758	A) 60,002 B) 53,248 C) 53,238 D) 53,148 E) none of these	4)	16 x 6	F) 636 G) 336 H) 96 I) 10 J) none of these
5)	302 x 8	A) 2486 B) 2416 C) 2406 D) 406 E) none of these	6)	54 x67	F) 3718 G) 3658 H) 378 I) 324 J) none of these
7)	6215 x 47	A) 292,105 B) 292,005 C) 43,505 D) 2,486,525 E) none of these	8)	6075 x 423	F) 2,681,795 G) 2,569,725 H) 2,562,775 I) 2,555,725 J) none of these
9)	7003 x 406	A) 2,843,218 B) 28,043,218 C) 2,443,218 D) 322,018 E) none of these	10)	7)84	F) 6 G) 10 H) 10 r1 I) 12 J) none of these

11) $96 \div 5 =$

A) 10
B) 11 r5
C) 19
D) 19 r1
E) none of these

12) $7\overline{)8216}$

F) 112
G) 1000 r1
H) 1314 r7
I) 1173 r5
J) none of these

13) $3\overline{)4032}$

A) 1010 r3
B) 1034 r1
C) 1344
D) 1343 r2
E) none of these

14) $\frac{2}{7} + \frac{3}{7} =$

F) $\frac{5}{7}$
G) $\frac{19}{7}$
H) 19
I) $\frac{5}{14}$
J) none of these

15) $2\frac{3}{8} + \frac{2}{8} =$

A) $2\frac{5}{16}$
B) $2\frac{21}{8}$
C) $2\frac{5}{8}$
D) 23
E) none of these

16) $4\frac{1}{9} + 2\frac{7}{9} =$

F) $6\frac{26}{9}$
G) $\frac{32}{9}$
H) $6\frac{8}{18}$
I) $6\frac{8}{9}$
J) none of these

17) $4\frac{1}{11} + 3\frac{3}{11} + 6\frac{2}{11} =$

A) $13\frac{39}{11}$
B) $\frac{52}{11}$
C) $13\frac{6}{11}$
D) $13\frac{6}{33}$
E) none of these

18) $4\frac{3}{5} + 2\frac{4}{5} =$

F) $6\frac{7}{10}$
G) $\frac{23}{10}$
H) $7\frac{2}{5}$
I) $6\frac{17}{5}$
J) none of these

19) $\frac{5}{9} - \frac{1}{9} =$

A) $\frac{6}{18}$
B) $\frac{4}{9}$
C) $\frac{24}{9}$
D) $\frac{6}{9}$
E) none of these

20) $2\frac{7}{11} - \frac{3}{11} =$

F) $\frac{34}{11}$
G) $2\frac{4}{22}$
H) $2\frac{10}{11}$
I) $2\frac{32}{11}$
J) none of these

21) $8 - 5\frac{3}{4} =$

A) $3\frac{3}{4}$

B) $3\frac{1}{4}$

C) $2\frac{1}{4}$

D) $\frac{10}{4}$

E) none of these

22) $6\frac{3}{7} - 2\frac{6}{7} =$

F) $3\frac{4}{7}$

G) $3\frac{3}{7}$

H) $\frac{31}{7}$

I) $4\frac{3}{7}$

J) none of these

23) Change $\frac{4}{6}$ to its lowest terms.

A) $\frac{3}{4}$

B) $\frac{2}{3}$

C) $\frac{6}{7}$

D) $\frac{1}{6}$

E) none of these

24) Change $\frac{15}{18}$ to its lowest terms.

F) $\frac{5}{6}$

G) $\frac{12}{18}$

H) $\frac{5}{9}$

I) $\frac{15}{17}$

J) none of these

END OF PART 3

Appendix D

ARITHMETIC INFORMATION, FIFTH GRADE

Student Number_____

Following are three different sets of arithmetic questions. These questions will tell us what arithmetic you already know and what arithmetic you will learn during this school year. Some of the questions will be easy for you and some of the questions will contain arithmetic you have not had. Each question has four or five possible answers. Choose the answer you believe is correct and mark it on the answer sheet as the first three examples have been done.

1) If $4 + n = 9$, then $n = ?$.

A) 4
B) 5
C) 6
D) 7
E) none of these

Notice that "B" has been marked on the answer sheet for example 1 because 5 is the correct answer.

2) If $4 \times p = 16$, then $p = ?$.

F) 2
G) 3
H) 4
I) 5
J) none of these

Notice that "H" has been marked on the answer sheet for example 2 because 4 is the correct answer.

3) If $24 \div 3 = n$, then $n = ?$.

A) 2
B) 3
C) 4
D) 5
E) none of these

Notice that "E" has been marked on the answer sheet for example 3 because the correct answer, 8, is not given as a choice.

Good heavy pencil marks have been made in the correct spaces. This will be necessary when you mark your answers.

Now, do examples 4 and 5.

4) If $9 - p = 4$, then $p = ?$.

F) 2
G) 3
H) 4
I) 5
J) none of these

5) If $(1 + 2) + n = 7$, then $n = ?$.

A) 2
B) 3
C) 4
D) 5
E) none of these

You should have marked "I" as the correct answer for example 4. You should have marked "C" as the correct answer for example 5.

You are now ready to answer the questions as you have done the examples above.

PART 1

Following is a set of 43 questions which will tell us what arithmetic understandings you know. Do not use paper to solve any of these questions. You should solve each of them in your head and then mark your answer on the answer sheet. Do not write in this test booklet or on the answer sheet.

You will have 33 minutes to complete Part 1. Stop at the end of Part 1.

At this time, ask any questions you might have.

---

1) In the expression 427 x 638 x 546, how will the answer be changed if it is worked 546 x 638 x 427?

A) the answer will be less
B) the answer will be greater
C) the answer will be the same
D) can't tell until it is worked out

2) A mixed number such as $3\frac{3}{4}$ means

F) $3 \times \frac{3}{4}$

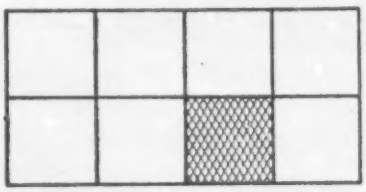
G) $3 + \frac{3}{4}$

H) $3 - \frac{3}{4}$

I) $3 \div \frac{3}{4}$

J) none of these

3) The shaded part of this figure is what fractional part of the figure?



A) $\frac{1}{2}$

B) $\frac{1}{3}$

C) $\frac{1}{4}$

D) $\frac{1}{5}$

E) none of these

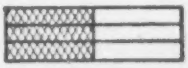
4) The inverse operation of division is

F) addition
G) subtraction
H) multiplication
I) division
J) none of these

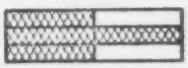
5) These statements are true: $4 + 6 = r$, $6 + 6 = s$, $4 + 4 = t$, $6 + 4 = u$. Which of the following is also true?

A) $r = s$
B) $r = t$
C) $r = u$
D) $s = u$
E) none of these

6) Which shaded figure shows one-half of one-third?

F) 

G)

H) 

I)

J) none of these

7) If 7 x t = 0, the "t" is always

A) zero
B) one
C) 7
D) ten
E) it is impossible to tell from the information given

8) The inverse operation of multiplication is

F) addition
G) subtraction
H) multiplication
I) division
J) none of these

9) As the denominator of a fractional number decreases and the numerator remains the same, the number

A) becomes larger
B) becomes smaller
C) remains the same
D) approaches one
E) can't tell from the information given

10) As the numerator of a fractional number decreases and the denominator remains the same, the number

F) becomes smaller
G) becomes larger
H) remains the same
I) gets close to one
J) can't tell from the information given

11) These statements are true: $a + b = d$, $c + b = e$, $c + c = f$.
Which of the following is also true?

A) $a = b + d$
B) $c = b + e$
C) $d + e = f$
D) $e = b + c$
E) none of these

12) If $0 \times y = 0$, the "y" is always

F) zero
G) one
H) two
I) ten
J) any number you choose

13) Which of the following is another numeral for 526

A) $(5 \times 10 \times 10) + (2 \times 10) + 6$
B) $(5 \times 5 \times 5) + (2 \times 2) + 6$
C) $(5 \times 100) + (2 \times 100) + 6$
D) $(5 \times 2 \times 6 \times 100) + (2 \times 6 \times 10) + 6$
E) none of these

14) A fraction such as $\frac{5}{7}$ means

F) choosing 7 parts after dividing an object into 5 parts
G) choosing 2 parts after dividing an object into 5 parts
H) choosing 5 parts after dividing an object into 7 parts
I) choosing 2 parts after dividing an object into 7 parts
J) none of these

15) If $a \times b = 0$, then

A) "a" always equals "b"
B) "b" must be zero
C) "a" must be zero
D) either "a" or "b" must be zero
E) none of these

16) Which fraction is the smallest?

F) $\frac{1}{3}$
G) $\frac{2}{3}$
H) $\frac{1}{4}$
I) $\frac{3}{4}$
J) $\frac{1}{2}$

17) If r x s = r, then "s" is always

A) zero
B) one
C) "r"
D) ten
E) it is impossible to tell from the information given

18) Jimmy's bike has a speedometer which shows miles and tenths of miles. It looks like this now | 027 | 7 |. How far will he ride before the speedometer reads | 033 | 0 |?

F) more than 5 miles
G) less than 5 miles
H) exactly 5 miles
I) can't tell from the information given

19) If r + s = r, then "s" is always

A) zero
B) one
C) "r"
D) ten
E) it is impossible to tell from the information given

20) To subtract in the exercise $\begin{array}{r} 645 \\ -162 \end{array}$, we should

F) make the 5 ones smaller
G) make the 5 ones larger
H) make the 4 tens smaller
I) make the 4 tens larger
J) none of these

21) If u - v = u, the "v" is always

A) zero
B) one
C) "u"
D) ten
E) it is impossible to tell from the information given

22) In the exercise $7\overline{)\,364}$, the 5 really stands for

F) 5 ones
G) 5 tens
H) 5 hundreds
I) 5 tenths
J) none of these

23) This line segment is cut into

A) sevenths
B) eighths
C) ninths
D) tenths
E) none of these

24) Four thousand three hundred seven is written

F) 4037
G) 0437
H) 4370
I) 40003007
J) none of these

25) Look at the numeral 85,626. The 6 on the left has a value how many times larger than the 6 on the right?

A) 100 times larger
B) 10 times larger
C) 1000 times larger
D) the same value
E) none of these

26) When multiplying in the problem $\begin{array}{r} 937 \\ \times\ 84 \\ \hline \end{array}$ we move the second partial product, which we get when we multiply by 8, one place to the left because

F) that is the rule in multiplying
G) the 8 means 8 tens
H) the answer must be larger than 937
I) the top number is a number larger than ten
J) none of these

27) Look at b ÷ a where "a" and "b" are both whole numbers greater than one. How does the answer compare with "b"?

A) the answer is greater than b
B) the answer is smaller than b
C) the answer is the same as b
D) can't tell until we see the whole number
E) can't tell until the division is done

28) Three-thirds plus four-fourths is

F) seven-sevenths
G) twelve-twelfths
H) $\frac{7}{4}$
I) 2
J) none of these

29) Which of the following shows $2\frac{3}{4}$ in another form?

A) $2\frac{7}{4}$

B) $2\frac{3}{8}$

C) $1\frac{7}{4}$

D) $1\frac{3}{8}$

E) none of these

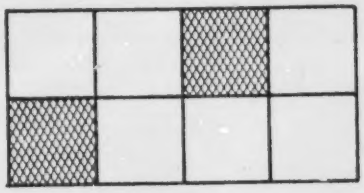
30) The shaded part of the figure is what part of the figure?

F) $\frac{1}{2}$

G) $\frac{1}{3}$

H) $\frac{1}{4}$

I) $\frac{1}{5}$



J) none of these

31) In the division example $463\overline{)5217468}$, the first figure in the quotient will be written in what column?

A) tens
B) hundreds
C) thousands
D) ten thousands
E) none of these

32) When finding the sum of several numbers of the same size, the operation that will give us the answer most quickly is

F) addition
G) subtraction
H) multiplication
I) division
J) none of these

33) Which of the following will give the same answer as 13 x 23?

A) (13 x 20) -3
B) (13 x 20) + 3
C) (13 x 20) + (13 x 3)
D) (10 x 20) + (3 x 3)
E) none of these

34) One-fourth of the set of x's to the right below is

$\left\{\begin{matrix} xxxxxx \\ \\ xxxxxx \end{matrix}\right\}$

F) xx
G) xxx
H) xxxx
I) xxxxx
J) none of these

35) Look at the problem r ÷ s. If "s" is the identity number for this division, then "s" is equal to

A) zero
B) one
C) ten
D) there isn't one
E) none of these

36) $\frac{6}{8} \div 1$ equals which of the following?

F) $\frac{6}{4}$
G) $\frac{3}{8}$
H) $\frac{3}{4}$
I) $\frac{5}{7}$
J) none of these

37) To reduce a fraction to lowest terms we

A) divide the numerator by the denominator
B) divide the denominator by the numerator
C) divide the numerator and the denominator by zero
D) divide the numerator and the denominator by a common divisor
E) none of these

38) What numeral is the same as ten and one-tenth?

F) 100.10
G) 100.01
H) 10.010
I) 10.01
J) none of these

39) Look at u x v where "u" and "v" are both whole numbers greater than one. How does the answer compare with "v"?

A) the answer is greater than v
B) the answer is smaller than v
C) the answer is the same as v
D) can't tell until I see the whole numbers
E) can't tell until I do the multiplication

40) Which fractional number is between 2 and 3?

F) $\frac{3}{2}$

G) $\frac{2}{3}$

H) $\frac{11}{5}$

I) $\frac{13}{4}$

J) none of these

41) Look at the problem u + v where "u" and "v" are both whole numbers greater than zero. If their sum is an odd number, then

A) both u and v are even numbers
B) both u and v are odd numbers
C) one number is even and one number is odd
D) v is always twice as large as u
E) none of these

42) The one in the numeral .0513 is in the

F) ones place
G) tenths place
H) hundredths place
I) thousandths place
J) none of these

43) To find the answer to $34\overline{)238}$ we could

A) multiply the answer and 34
B) divide 8 by 4
C) add 238 thirth-four times and use the sum as the answer
D) find out how many 34's can be subtracted from 238 and use this number as the answer
E) none of these

END OF PART 1

DO NOT TURN THE PAGE

PART 2

Following is a set of 13 problems which will tell us what arithmetic problems you can work.

You may use paper to solve these problems. Put three clean sheets of paper on your desk to figure on. Do not write in this test booklet or on the answer sheet.

You will have 22 minutes to complete Part 2. Do not go on to Part 3 until you are instructed to do so.

At this time, ask any questions you might have.

---

1) The grocer had 137 pounds of apples. At the end of the day he had 49 pounds of apples. How many pounds of apples did he sell during the day?

A) 111 pounds
B) 78 pounds
C) 88 pounds
D) 98 pounds
E) none of these

---

2) After earning 35¢ a day for 14 days Joe still needs $2.47 to buy a present. How much does the present cost?

F) $4.90
G) $ .49
H) $2.82
I) $7.37
J) none of these

---

3) Jane had a piece of ribbon 4 yards long. She cut it into 8 pieces of equal length. What was the measure in inches of each piece?

A) 72 inches
B) 18 inches
C) 16 inches
D) 32 inches
E) none of these

---

4) Frank had 17 models at the beginning of the year. Now he has 36 models. At $1.35 each what is the value of the models added to his collection?

F) $25.65
G) $48.60
H) $26.65
I) $71.55
J) none of these

---

5) At the club picnic there were 7 gallons of ice cream. If one quart of ice cream serves 8 people, how many people can be served?

A) 56 people
B) 112 people
C) 168 people
D) 224 people
E) none of these

6) The Boys Club collected $9.80 selling popcorn, at 5 cents a bag. How many bags of popcorn did they sell?

F) 4900 bags
G) 110 bags
H) 196 bags
I) 112 bags
J) none of these

---

7) Jane made scores of 25, 22, 18, 21, 15, and 25 on arithmetic tests. What was Jane's average score?

A) 126 points
B) 105 points
C) 21 points
D) 15 points
E) none of these

---

8) Jane bought 4 records at $1.89 each. The tax on all 4 records together was 34¢. How much change should she get from $10.00?

F) $1.08
G) $2.10
H) $2.44
I) $7.56
J) none of these

---

9) Jim has four small rabbits. They weigh $12\frac{1}{3}$ oz., $13\frac{3}{4}$ oz., $11\frac{7}{8}$ oz., and $10\frac{5}{6}$ oz. How much do all four rabbits weigh?

A) $48\frac{19}{24}$ oz.
B) $48\frac{1}{8}$ oz.
C) $48\frac{7}{12}$ oz.
D) $49\frac{5}{24}$ oz.
E) none of these

---

10) Phil had saved $8\frac{3}{4}$ dollars. He earned $4\frac{1}{2}$ dollars. He then spent $9\frac{1}{4}$ dollars. How much did he have left?

F) $3.00
G) $3.50
H) $4.00
I) $4.50
J) none of these

---

11) Mr. Smith needs 1200 sq. ft. of storage space. He rented one building that was 35 ft. by 27 ft. How many more sq. ft. of space does he need?

A) 945 sq. ft.
B) 255 sq. ft.
C) 260 sq. ft.
D) 23 ft. by 27 ft. more
E) none of these

12) Jean, Jane, and Mary weigh $194\frac{1}{4}$ lbs. together. If Jane weighs $60\frac{1}{2}$ lbs. and Jean weighs $55\frac{1}{4}$ lbs., how much does Mary weigh?

F) $115\frac{3}{4}$ lbs.
G) $79\frac{3}{4}$ lbs.
H) 79 lbs.
I) $78\frac{1}{2}$ lbs.
J) none of these

---

13) Mr. Smith drove 456 miles in 8 hours. He expects to drive 3 more hours at the same average speed. How many more miles does he expect to drive?

A) 24 miles
B) 1368 miles
C) 459 miles
D) 171 miles
E) none of these

END OF PART 2

DO NOT TURN THE PAGE

PART 3

Following is a set of 23 problems which will tell us what arithmetic problems you can work.

You may use paper to solve these problems. Put three clean sheets of paper on your desk to figure on. Do not figure in this test booklet or on the answer sheet.

You will have 28 minutes to complete Part 3.

At this time, ask any question you might have.

---

1) $\begin{array}{r} 7003 \\ \times\ 438 \\ \hline \end{array}$

A) 3,067,314
B) 3,057,314
C) 2,968,314
D) 256,114
E) none of these

2) $6\overline{)4302}$

F) 700 r102
G) 850 r2
H) 717
I) 1203
J) none of these

3) $56\overline{)336,896}$

A) 601 r840
B) 616
C) 5305 r36
D) 6016
E) none of these

4) $302\overline{)4,173,057}$

F) 13,453 r251
G) 1361 r386
H) 13,818 r21
I) 1411 r302
J) none of these

5) Change $\frac{16}{64}$ to its lowest terms

A) 4
B) $\frac{61}{46}$
C) $\frac{1}{4}$
D) $\frac{8}{8}$
E) none of these

6) Change $\frac{21}{35}$ to its lowest terms

F) $\frac{3}{5}$
G) $\frac{1}{3}$
H) $\frac{12}{53}$
I) $1\frac{2}{5}$
J) none of these

7) Change $\frac{19}{5}$ to a mixed number in lowest terms

A) $\frac{91}{5}$
B) $8\frac{3}{5}$
C) $3\frac{4}{5}$
D) $1\frac{4}{5}$
E) none of these

8) Change $4\frac{7}{5}$ to an improper fraction

F) $4\frac{7}{5}$
G) $5\frac{2}{5}$
H) $4\frac{1}{1}$
I) $4\frac{5}{7}$
J) none of these

9) $\frac{5}{6} = \frac{}{42}$

A) 30
B) 35
C) 41
D) 5
E) none of these

10) $\frac{1}{7} + \frac{3}{7} =$

F) $\frac{2}{7}$
G) $\frac{4}{7}$
H) $\frac{4}{14}$
I) $\frac{4}{49}$
J) none of these

11) $\frac{3}{4} + \frac{5}{8} =$

A) $\frac{8}{8}$
B) $\frac{8}{12}$
C) $\frac{1}{2}$
D) $1\frac{3}{8}$
E) none of these

12) $\frac{1}{3} + \frac{3}{5} + \frac{5}{6} =$

F) $1\frac{23}{30}$
G) $1\frac{13}{30}$
H) $\frac{9}{30}$
I) $\frac{9}{14}$
J) none of these

13) $\frac{5}{8} - \frac{1}{4} =$

A) $\frac{4}{32}$
B) $\frac{3}{8}$
C) $\frac{5}{8}$
D) $\frac{4}{4}$
E) none of these

14) $\frac{5}{6} - \frac{1}{4} =$

F) $\frac{1}{3}$
G) $\frac{7}{12}$
H) $1\frac{1}{12}$
I) $\frac{4}{2}$
J) none of these

15) $4 - \frac{3}{5} =$

A) $4\frac{3}{5}$
B) $3\frac{3}{5}$
C) $3\frac{2}{5}$
D) $\frac{2}{5}$
E) none of these

16) $24\frac{4}{15} - 6\frac{9}{10}$

F) $18\frac{11}{30}$
G) $18\frac{21}{30}$
H) $17\frac{1}{30}$
I) $17\frac{11}{30}$
J) none of these

17) $10\frac{1}{11} - 4\frac{3}{5}$

A) $5\frac{27}{55}$
B) $4\frac{80}{55}$
C) $5\frac{8}{55}$
D) $6\frac{33}{55}$
E) none of these

18) $2.3 + 4.6 =$

F) 30
G) 6.9
H) 8.9
I) 8.18
J) none of these

19) $8.4 + .4 =$

A) 26
B) 12.4
C) 8.8
D) 8.0
E) none of these

20) $15.63 + 4.72 + 2.5 =$

F) 21.14
G) 22.85
H) 21.185
I) 21.141
J) none of these

21) $8.62 - 6.41 =$

A) 730.116
B) 2.21
C) 230
D) 14.103
E) none of these

22) $22.74 - 9.8 =$

F) 21.86
G) 75.26
H) 12.94
I) 13.94
J) none of these

23) $14.21 - .4 =$

A) 14.23
B) 10.21
C) 14.17
D) 13.81
E) none of these

END OF PART 3

Appendix E

ARITHMETIC INFORMATION, SIXTH GRADE

Student Number_____

Following are three different sets of arithmetic questions. These questions will tell us what arithmetic you already know and what arithmetic you will learn during this school year. Some of the questions will be easy for you and some of the questions will contain arithmetic you have not had. Each question has four or five possible answers. Choose the answer you believe is correct and mark it on the answer sheet as the first three examples have been done.

1) If $4 + n = 9$, then $n = ?$.

A) 4
B) 5
C) 6
D) 7
E) none of these

Notice that "B" has been marked on the answer sheet for example 1 because 5 is the correct answer.

2) If $4 \times p = 16$, then $p = ?$.

F) 2
G) 3
H) 4
I) 5
J) none of these

Notice that "H" has been marked on the answer sheet for example 2 because 4 is the correct answer.

3) If $24 \div 3 = n$, then $n = ?$.

A) 2
B) 3
C) 4
D) 5
E) none of these

Notice that "E" has been marked on the answer sheet for example 3 because the correct answer, 8, is not given as a choice.

Good heavy pencil marks have been made in the correct spaces. This will be necessary when you mark your answers.

Now, do examples 4 and 5.

4) If $9 - p = 4$, then $p = ?$.

F) 2
G) 3
H) 4
I) 5
J) none of these

5) If $(1 + 2) + n = 7$, then $n = ?$.

A) 2
B) 3
C) 4
D) 5
E) none of these

You should have marked "I" as the correct answer for example 4. You should have marked "C" as the correct answer for example 5.

You are now ready to answer the questions as you have done the examples above.

PART 1

Following is a set of 44 questions which will tell us what arithmetic understandings you know. Do not use paper to solve any of these questions. You should solve each of them in your head and then mark your answer on the answer sheet. Do not write in this test booklet or on the answer sheet.

You will have 34 minutes to complete Part 1. Stop at the end of Part 1. Do not go on to Part 2 until you are instructed to do so.

At this time, ask any questions you might have.

---

1) As the denominator of a fractional number decreases and the numerator remains the same, the number

A) becomes larger
B) becomes smaller
C) remains the same
D) approaches one
E) can't tell from the information given

2) As the numerator of a fractional number decreases and the denominator remains the same, the number

F) becomes smaller
G) becomes larger
H) remains the same
I) gets close to one
J) can't tell from the information given

3) These statements are true: $a + b = d$, $c + b = e$, $c + c = f$. Which of the following is also true?

A) $a = b + d$
B) $c = b + e$
C) $d + e = f$
D) $e = b + c$
E) none of these

4) If $0 \times y = 0$, then "y" is always

F) zero
G) one
H) two
I) ten
J) any number you choose

5) Which of the following is another numeral for 526?

A) $(5 \times 10 \times 10) + (2 \times 10) + 6$
B) $(5 \times 5 \times 5) + (2 \times 2) + 6$
C) $(5 \times 100) + (2 \times 100) + 6$
D) $(5 \times 2 \times 6 \times 100) + (2 \times 6 \times 10) + 6$
E) none of these

6) A fraction such as $\frac{5}{7}$ means

F) choosing 7 parts after dividing an object into 5 parts
G) choosing 2 parts after dividing an object into 5 parts
H) choosing 5 parts after dividing an object into 7 parts
I) choosing 2 parts after dividing an object into 7 parts
J) none of these

7) If $a \times b = 0$, then

A) "a" always equals "b"
B) "b" must be zero
C) "a" must be zero
D) either "a" or "b" must be zero
E) none of these

8) Which fraction is the smallest?

F) $\frac{1}{3}$

G) $\frac{2}{3}$

H) $\frac{1}{4}$

I) $\frac{3}{4}$

J) $\frac{1}{2}$

9) If r x s = r, then "s" is always

A) zero
B) one
C) "r"
D) ten
E) it is impossible to tell from the information given

10) Jimmy's bike has a speedometer which shows miles and tenths of miles. It looks like this now | 027 | 7 |. How far will he ride before the speedometer reads | 033 | 0 |?

F) more than 5 miles
G) less than 5 miles
H) exactly 5 miles
I) can't tell from the information given

11) If r + s = r, then "s" is always

A) zero
B) one
C) "r"
D) ten
E) it is impossible to tell from the information given

12) To subtract in the exercise $\begin{array}{r} 645 \\ -162 \end{array}$, we should

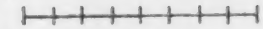
F) make the 5 ones smaller
G) make the 5 ones larger
H) make the 4 tens smaller
I) make the 4 tens larger
J) none of these

13) If u - v = u, then "v" is always

A) zero
B) one
C) "u"
D) ten
E) it is impossible to tell from the information given

14) In the exercise $7\overline{)364}$ (5), the 5 really stands for

F) 5 ones
G) 5 tens
H) 5 hundreds
I) 5 tenths
J) none of these

15) This line segment is cut into 

A) sevenths
B) eighths
C) ninths
D) tenths
E) none of these

16) Four thousand three hundred seven is written

F) 4037
G) 0437
H) 4370
I) 40003007
J) none of these

17) Look at the numeral 85,626. The 6 on the left has a value how many times larger than the 6 on the right?

A) 100 times larger
B) 10 times larger
C) 1000 times larger
D) the same value
E) none of these

18) When multiplying in the probelm $\begin{array}{r} 937 \\ \times\ 84 \\ \hline \end{array}$ we move the second partial product, which we get when we multiply by 8, one place to the left because

F) that is the rule in multiplying
G) the 8 means 8 tens
H) the answer must be larger than 937
I) the top number is a number larger than ten
J) none of these

19) Which of the following will give the same answer as 13 x 23?

A) (13 x 20) - 3
B) (13 x 20) + 3
C) (13 x 20) + (13 x 3)
D) (10 x 20) + (3 x 3)
E) none of these

20) One-fourth of the set of x's to the right is

$$\left\{\begin{matrix} xxxxxx \\ \\ xxxxxx \end{matrix}\right\}$$

F) xx
G) xxx
H) xxxx
I) xxxxx
J) none of these

21) Look at the problem r ÷ s. If "s" is the identity number for this division, then "s" is equal to

A) zero
B) one
C) ten
D) there isn't one
E) none of these

22) $\frac{6}{8} \div 1$ equals which of the following?

F) $\frac{6}{4}$
G) $\frac{3}{8}$
H) $\frac{3}{4}$
I) $\frac{5}{7}$
J) none of these

23) To reduce a fraction to the lowest terms we

A) divide the numerator by the denominator
B) divide the denominator by the numerator
C) divide the numerator and the denominator by zero
D) divide the numerator and the denominator by a common divisor
E) none of these

24) What numeral is the same as ten and one-tenth?

F) 100.10
G) 100.01
H) 10.010
I) 10.01
J) none of these

25) Look at u x v where "u" and "v" are both whole numbers greater than one. How does the answer compare with "v"?

A) the answer is greater than v
B) the answer is smaller than v
C) the answer is the same as v
D) can't tell until I see the whole numbers
E) can't tell until I do the multiplication

26) Which fractional number is between 2 and 3?

F) $\frac{3}{2}$

G) $\frac{2}{3}$

H) $\frac{11}{5}$

I) $\frac{13}{4}$

J) none of these

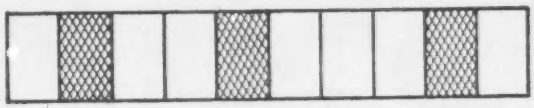
27) Look at the problem u + v where "u" and "v" are both whole numbers greater than zero. If their sum is an odd number, then

A) both u and v are even numbers
B) both u and v are odd numbers
C) one number is even and one number is odd
D) v is always twice as large as u
E) none of these

28) The one in the numeral .0513 is in the

F) ones place
G) tenths place
H) hundredths place
I) thousandths place
J) none of these

29) What part of the figure is shaded?



A) 0.03
B) 3.00
C) 0.07
D) 7.00
E) none of these

30) What does h% mean?

F) 100 x h

G) $\frac{h}{100}$

H) $\frac{100}{h}$

I) divide "h" into 100 parts
J) none of these

31) To find the answer to $34\overline{)238}$ we could

A) multiply the answer and 34
B) divide 8 by 4
C) add 238 thirty-four times and use the sum of the answer
D) find out how many 34's can be subtracted from 238 and use this number as the answer
E) none of these

32) Which number is larger than 4.035?

F) 4.034
G) 4.029
H) 4.1
I) 4.000
J) none of these

33) To obtain the answer to the exercise $\begin{array}{r} 754 \\ +163 \\ \hline \end{array}$, we must change the form of one number. What number is it?

A) 7 ones
B) 11 tens
C) 8 hundreds
D) 9 hundreds
E) none of these

34) If "a" represents an odd number, the next larger odd number can be represented as

F) 2 x a
G) (2 x a) + 1
H) a + 1
I) a + 2
J) none of these

35) If "u" is any number different from zero, then u ÷ u is equal to

A) zero
B) one
C) ten
D) "u"
E) none of these

36) How many even whole numbers are there between 35 and 39?

F) none
G) one
H) two
I) three
J) none of these

37) Which fraction is the largest?

A) $\frac{1}{3}$

B) $\frac{2}{3}$

C) $\frac{1}{4}$

D) $\frac{3}{4}$

E) $\frac{1}{2}$

38) Look at the problem a x $\frac{b}{c}$ where "a" is a whole number, bigger than zero and $\frac{b}{c}$ is a proper fraction. How does the answer compare with the whole number "a"?

F) the answer is larger than the whole number
G) the answer is smaller than the whole number
H) the answer is the same as the whole number
I) we can't tell until we see the numbers
J) we can't tell until the problem is worked

39) Look at the problem a ÷ $\frac{c}{b}$ where "a" is a whole number bigger than zero and $\frac{c}{b}$ is an improper fraction different from one. How does the answer compare with the whole number "a"?

A) the answer is larger than the whole number
B) the answer is smaller than the whole number
C) the answer is the same as the whole number
D) we can't tell until we see the numbers
E) we can't tell until the problem is worked

40) How would the answer to $\begin{array}{r} 427 \\ \times\ 58 \end{array}$ be changed if we changed 427 to 4270 and 58 to 5.8?

F) the new answer would be the same as the old answer
G) the new answer would be one-tenth as large as the old answer
H) the new answer would be ten times larger than the old answer
I) the new answer would be one hundred times larger than the old answer
J) none of these

41) How would the answer to $\begin{array}{r} 74.90 \\ \times\ 65.8 \\ \hline \end{array}$ be changed if we removed the zero from 74.90?

A) the new answer would be equal to the old answer
b) the new answer would be one-tenth as large as the old answer
C) the new answer would be one-hundredth as large as the old answer
D) the new answer would be larger than the old answer because there would be fewer decimal places in the new answer
E) we can't tell until we do the multiplication

42) How would the answer to $950\overline{)63650}$ be changed if the zeros in the two numbers were removed?

F) the new answer would be equal to the old answer
G) the new answer would be one hundred times larger than the old answer
H) the new answer would be one-hundredth as large as the old answer
I) the new answer would be ten times larger than the old answer
J) none of these

43) Which number is smaller than 2.047?

A) 2.111
B) 2.048
C) 2.050
D) 2.1
E) none of these

44) How does the answer to $\begin{array}{r} 658 \\ \times\ 39 \\ \hline \end{array}$ compare with 658?

F) the answer is ten times larger than 658
G) the answer is thirty times larger than 658
H) the answer is 658 times larger than 658
I) the answer is 39 times larger than 658
J) none of these

END OF PART 1

DO NOT TURN THE PAGE

PART 2

Following is a set of 16 problems which will tell us what arithmetic problems you can work.

You may use paper to solve these problems. Put three clean sheets of paper on your desk to figure on. Do not write in this test booklet or on the answer sheet.

You will have 27 minutes to complete Part 2. Do not go on to Part 3 until you are instructed to do so.

At this time, ask any questions you might have.

---

1) After delivering its $3\frac{1}{4}$ ton load of hay, a truck weighed 7256 pounds. How much did the truck and its load weigh?

A) 756 lbs.
B) $7,259\frac{1}{4}$ lbs.
C) 13,756 lbs.
D) 10,500 lbs.
E) none of these

---

2) A hay dealer sold 124 tons of hay in October, 133 tons in November, 147 tons in December, 156 tons in January, and 142 tons in February. What was the average amount of hay sold per month?

F) $14\frac{2}{5}$ tons
G) 110 tons
H) $136\frac{2}{5}$ tons
I) $140\frac{2}{5}$ tons
J) none of these

---

3) Joe walks $\frac{2}{3}$ of a mile to school. Frank walks $\frac{1}{4}$ as far. How many <u>feet</u> does Frank walk to school?

A) 100 feet
B) 440 feet
C) 880 feet
D) 4840 feet
E) none of these

---

4) Mr. Carpenter, the grocer, had $12\frac{2}{3}$ lbs. of peanuts. He put them into bags containing $\frac{1}{3}$ pound each. How many bags of peanuts did he have?

F) 5 bags
G) 36 bags
H) 38 bags
I) 89 bags
J) none of these

5) Mr. Jones drove his truck 385 miles in 7 hours. The maximum speed limit was 60 miles per hour. How much below the maximum speed was his average speed?

A) 3 miles per hour
B) 5 miles per hour
C) 10 miles per hour
D) 55 miles per hour
E) none of these

---

6) Jack, a race car driver, averaged 93 miles per hour in a 400 mile race. To the nearest tenth of an hour how long did it take Jack to complete the race?

F) 4 hours 28 minutes
G) 4.3 hours
H) 4 hours
I) 2 hours
J) none of these

---

7) Mrs. Smith uses 12 cans of water with 3 cans of frozen juice. If each can holds 6 ounces, how many ounces of water will she use with 5 cans of frozen juice?

A) 20 oz.
B) 30 oz.
C) 120 oz.
D) 150 oz.
E) none of these

---

8) The speed limit in a German city is 40 kilometers per hour. To the nearest tenth in miles per hour, what is the speed limit in that city? (.62 mi = 1 km)

F) 2.4 miles per hour
G) 24.8 miles per hour
H) 55 miles per hour
I) 70 miles per hour
J) none of these

---

9) Joe delivered 85% of his 160 papers. How many papers did he deliver?

A) 120 papers
B) 135 papers
C) 136 papers
D) 188 papers
E) none of these

---

10) Jane spelled 90% of the words correctly. She spelled 18 words correctly. How many words were on the test?

F) 19 words
G) 20 words
H) 28 words
I) 108 words
J) none of these

---

11) Jean took a spelling test of 80 words. She spelled 64 of the words correctly. What percentage of the words did she spell correctly?

A) 16%
B) 64%
C) 80%
D) 85%
E) none of these

12) Joe took a 40 problem arithmetic test. He correctly worked 80% of the problems. How many of the problems did he miss?

F) 5 problems
G) 8 problems
H) 10 problems
I) 20 problems
J) none of these

---

13) In Woodland School 8% of the pupils made a perfect score on an arithmetic test. If 22 pupils made a perfect score, how many pupils attend Woodland School?

A) 176 students
B) 200 students
C) 275 students
D) 2024 students
E) none of these

---

14) A basketball team lost 40% of the 25 games it played. How many games did the team win?

F) 10 games
G) 11 games
H) 15 games
I) 20 games
J) none of these

---

15) The Jones family had an income of $6500. They saved $600 of this money. To the nearest whole percent, what percent of the income did they save?

A) 59%
B) 50%
C) 10.5%
D) 9%
E) none of these

---

16) When buying a new car Mr. Woods paid a sales tax of $4\frac{1}{2}\%$ on the purchase price.

The purchase price was $4000. How much sales tax did Mr. Woods pay?

F) $160
G) $170
H) $180
I) $888.89
J) none of these

END OF PART 2

DO NOT TURN THE PAGE

PART 3

Following is a set of 30 problems which will tell us what arithmetic problems you can work.

You may use paper to solve these problems. Put three clean sheets of paper on your desk to figure on. Do not figure in this test booklet or on the answer sheet.

You will have 36 minutes to complete Part 3.

At this time, ask any questions you might have.

---

1) $\begin{array}{r} 80402 \\ \times\ 728 \\ \hline \end{array}$

A) 58,732,656
B) 58,532,656
C) 58,539,656
D) 56,092,656
E) none of these

2) $42\overline{)91358}$

F) 2175 r8
G) 2079 r40
H) 2314
I) 2172 r34
J) none of these

3) $409\overline{)46782}$

A) 114 r156
B) 111 r283
C) 1110 r283
D) 114 r56
E) none of these

4) $\frac{1}{3} + \frac{3}{5} + \frac{5}{6} =$

F) $\frac{9}{14}$
G) $\frac{9}{30}$
H) 1
I) $1\frac{23}{30}$
J) none of these

5) $7\frac{3}{4}$
$4\frac{1}{2}$
$2\frac{9}{10}$
$3\frac{1}{3}$

A) $16\frac{14}{19}$
B) $16\frac{14}{60}$
C) $18\frac{27}{60}$
D) $18\frac{9}{20}$
E) none of these

6) $4 - \frac{7}{11} =$

F) $4\frac{7}{11}$
G) $4\frac{4}{11}$
H) $3\frac{7}{11}$
I) $3\frac{4}{11}$
J) none of these

7) $24\frac{4}{15} - 6\frac{9}{10}$

A) $17\frac{11}{30}$
B) $18\frac{5}{25}$
C) $18\frac{5}{5}$
D) $17\frac{1}{30}$
E) none of these

8) $10\frac{1}{11} - 4\frac{3}{5}$

F) $6\frac{2}{6}$
G) $6\frac{2}{55}$
H) $5\frac{27}{55}$
I) $5\frac{5}{10}$
J) none of these

9) $\frac{9}{10} \div \frac{5}{6} =$

A) $\frac{54}{60}$
B) $\frac{50}{54}$
C) $\frac{3}{4}$
D) $\frac{1}{3}$
E) none of these

10) $\frac{1}{2} \times 10 =$

F) $\frac{1}{5}$
G) 5
H) $\frac{1}{20}$
I) 20
J) none of these

11) $4\frac{1}{2} \times 3\frac{2}{3} =$

A) $12\frac{2}{6}$
B) $\frac{22}{27}$
C) 15
D) $16\frac{1}{2}$
E) none of these

12) $\frac{1}{2} \times 3\frac{2}{3} \times \frac{8}{11} =$

F) $3\frac{16}{66}$
G) $1\frac{5}{6}$
H) $2\frac{2}{3}$
I) $1\frac{1}{3}$
J) none of these

13) $\frac{3}{4} \div \frac{5}{7} =$

A) $\frac{15}{28}$
B) $1\frac{1}{20}$
C) $\frac{20}{21}$
D) $2\frac{11}{12}$
E) none of these

14) $10 \div \frac{1}{4} =$

F) 40
G) $\frac{1}{40}$
H) $\frac{10}{4}$
I) $\frac{4}{10}$
J) none of these

15) $\frac{1}{4} \div 10 =$

A) 40
B) $\frac{1}{40}$
C) $\frac{10}{4}$
D) $\frac{4}{10}$
E) none of these

16) $4\frac{5}{6} \div 3\frac{4}{9} =$

F) $1\frac{1}{5}$
G) $1\frac{25}{62}$
H) $\frac{5}{6}$
I) $\frac{29}{31}$
J) none of these

17) 15.63 + 2.55 =

A) 13.08
B) 18.18
C) 36.18
D) 13.18
E) none of these

18) 4.62 + .403 + 27.2 =

F) 32.223
G) 11.37
H) 31.223
I) 12.12
J) none of these

19) 15.21 - 2.3 =

A) 13
B) 12.86
C) 17.58
D) 12.84
E) none of these

20) 201.3 - 48.004 =

F) 153.304
G) 721.34
H) 153.296
I) 720.34
J) none of these

21) 2.2 x 8 =

A) 1.76
B) 176
C) 16.16
D) .176
E) none of these

22) 13 x .43 =

F) 52.39
G) 52.29
H) 5.69
I) 5.59
J) none of these

23)
$$\begin{array}{r} 22.34 \\ \times\ 7.2 \\ \hline \end{array}$$

A) 160.848
B) 1608.48
C) 180.848
D) 1808.48
E) none of these

24)
$$\begin{array}{r} 42 \\ \times\ .0004 \\ \hline \end{array}$$

F) .000168
G) .0168
H) .00168
I) .168
J) none of these

25) To the nearest tenth

$4.31\overline{)71.216}$

A) 16.0
B) 16.5
C) 16.6
D) 16.7
E) none of these

26) $.005\overline{)47.2}$

F) 94500
G) 945
H) 94.5
I) 9450
J) none of these

27) To the nearest tenth of a percent 57 = ____% of 112

A) 50.9%
B) 50%
C) 19.9%
D) 19.8%
E) none of these

28) 192 is what percent of 256?

F) 13%
G) 14%
H) 75%
I) 15%
J) none of these

29) 45% = what fraction in lowest terms?

A) $\frac{4}{5}$
B) $\frac{5}{4}$
C) $\frac{9}{20}$
D) $\frac{1}{125}$
E) none of these

30) 58% = what fraction in lowest terms?

F) $\frac{5}{8}$
G) $\frac{29}{50}$
H) $\frac{8}{5}$
I) $\frac{1}{160}$
J) none of these

END OF PART 3

Appendix F

TEACHER QUESTIONNAIRE

Please complete the following questionnaire and return it to your principal. All hours are quarter hours. If your courses were semester hours, use the table at the right to convert to quarter hours. Circle the appropriate response.

Semester Hours	Quarter Hours
3	$4\frac{1}{2}$
4	6
5	$7\frac{1}{2}$
6	9
7	$10\frac{1}{2}$
8	12
9	$13\frac{1}{2}$
10	15
11	$16\frac{1}{2}$
12	18
13	$19\frac{1}{2}$
14	21

Teacher number _____

1) Number of quarter hours of college mathematics courses, including inservice salary credit course. (Do not include methods courses in mathematics.)

none 1 to 7 7 to 13 13 to 19 19 or more

Were any of these courses calculus? yes no

How many of these quarter hours were new mathematics?

none 1 to 7 7 to 13 13 to 19 19 or more

2) How long ago was the last of these mathematics courses taken?

past year 1 to 2 years 2 to 5 years

5 to 10 years 10 or more years

3) Number of quarter hours concerned with the teaching of mathematics. (Methods courses in the teaching of arithmetic or mathematics. Do not include courses counted in 1) above.)

none 1 to 4 4 to 9 9 to 13 13 or more

4) How long ago was the last of these methods courses taken?

past year 1 to 2 years 2 to 5 years 5 to 10 years

10 or more years

5) Number of quarter hours of professional education courses. (This includes all education courses and some psychology courses. A close approximation is sufficient.)

0 to 20 20 to 30 30 to 40 40 to 50 50 or more

6) Number of years of teaching experience. (Do not count this year or substituting experience.)

none 1 2 3 or 4 5 or 6 7 to 10 10 to 15

15 to 20 20 or more

7) Number of years of teaching experience in the Spokane District. (Do not count this year or substituting experience.)

none 1 2 3 or 4 5 or 6 7 to 10 10 to 15

15 to 20 20 or more

Appendix G

PRINCIPAL'S RATING SHEET

Rate the teacher numbered on a scale of 1 to 7. A rating of 1 indicates a poor rating and a rating of 7 indicates an excellent rating. Circle the appropriate rating. Complete and return with your answer sheets.

Teacher number _____

1) Considering all aspects of teaching, this teacher is:

1 2 3 4 5 6 7

2) Considering all aspects of arithmetic teaching, this teacher is:

1 2 3 4 5 6 7

3) Considering the newer methods and ideas in arithmetic, this teacher believes in and uses these ideas:

1 2 3 4 5 6 7

VITA

Demitrios P. Prekeges was born in Cheyenne, Wyoming, on June 28, 1930, the son of Paraskevi A. and Peter G. Prekeges. After graduating from Warden High School, Warden, Washington, in 1947, he attended Eastern Washington State College, Cheney, Washington, from which he received the degree of Bachelor of Arts in Education in June, 1951. In August, 1963, he was awarded the degree of Master of Arts in the Teaching of Mathematics from the University of Montana, Missoula, Montana.

From 1951 to 1962, he served as an elementary school teacher, junior high school teacher, and senior high school teacher in Warden, Wishram, Cle Elum, and Mead Schools, all in the State of Washington. He spent 1962-1963 at The Ohio State University with a National Science Foundation Academic Year Institute grant. From 1963 to the present he has been a member of the Mathematics Department at Eastern Washington State College, Cheney, Washington. His major duty is teacher training.

In 1953, he married Marilyn Gertrude Daugherty. They have four sons: Peter, born July 20, 1956; David, born January 17, 1961; Donald, born October 31, 1962; and Paul, born May 10, 1966.

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